Comparison of Random Walk based techniques for estimating network averages

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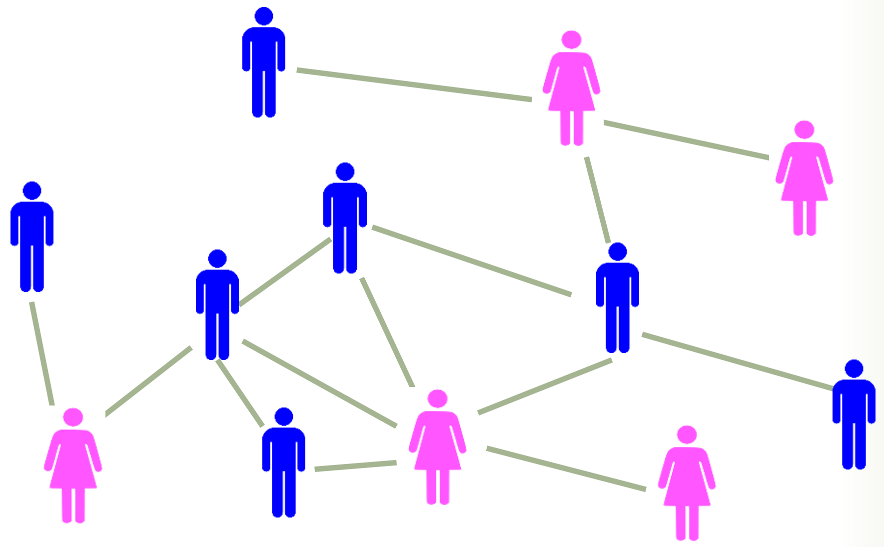
Motivation

- Estimation in Online Social Network (OSN)
- Example:
  What proportion of a population supports a given political party?

  How young a given social network is?



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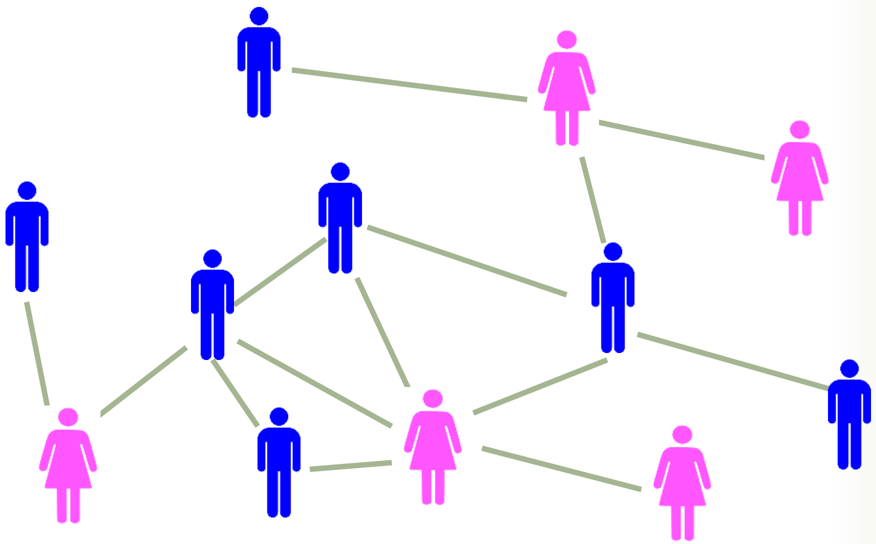

Easy to answer if the graph is fully known beforehand

What if the network is not known?

- Can only crawl network
- Few queries

Problem definition

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Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

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Estimate $\mu(\mathcal{G}) = \frac{1}{|\mathcal{V}|} \sum_{u \in \mathcal{V}} f(u)$

- Graph is unknown
- Only local information available

$\left\{ \begin{array}{l} \text{Seed nodes and their neighbor IDs} \\ \text{Query (visit) a neighbor} \\ \text{Visited nodes and their neighbor IDs} \end{array} \right.$

Random Walk based estimation

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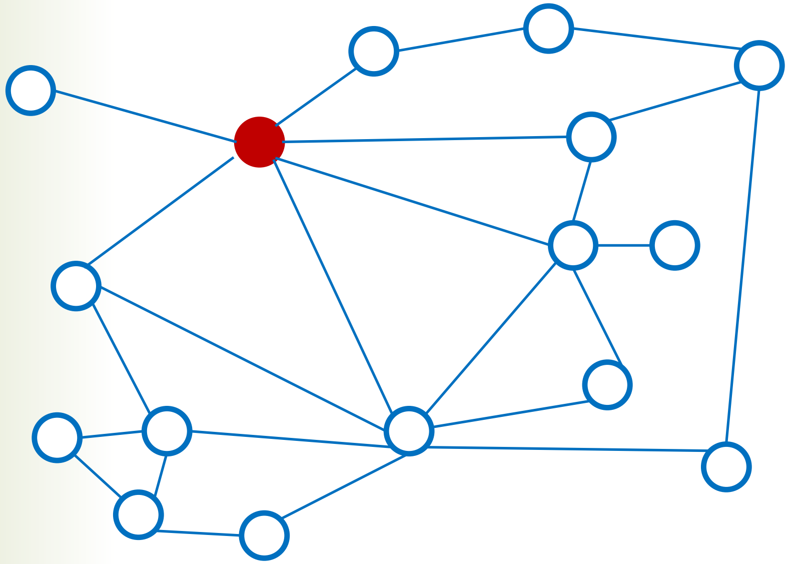

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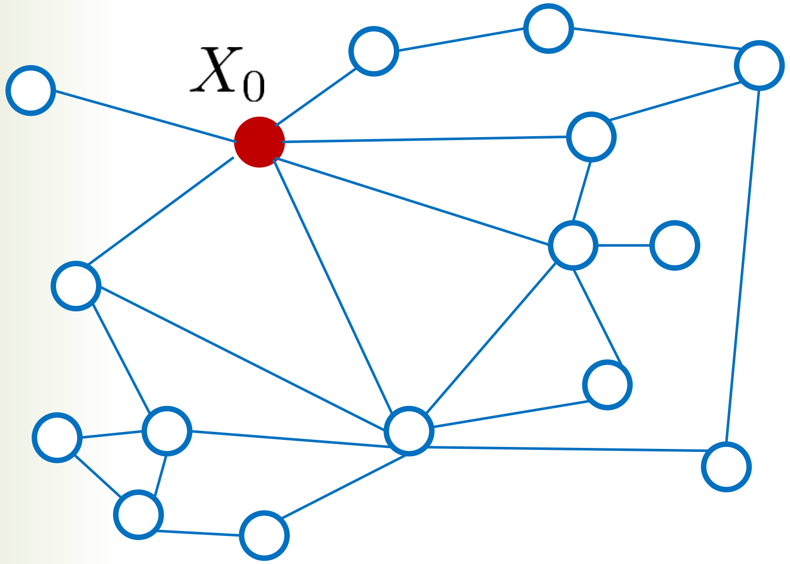

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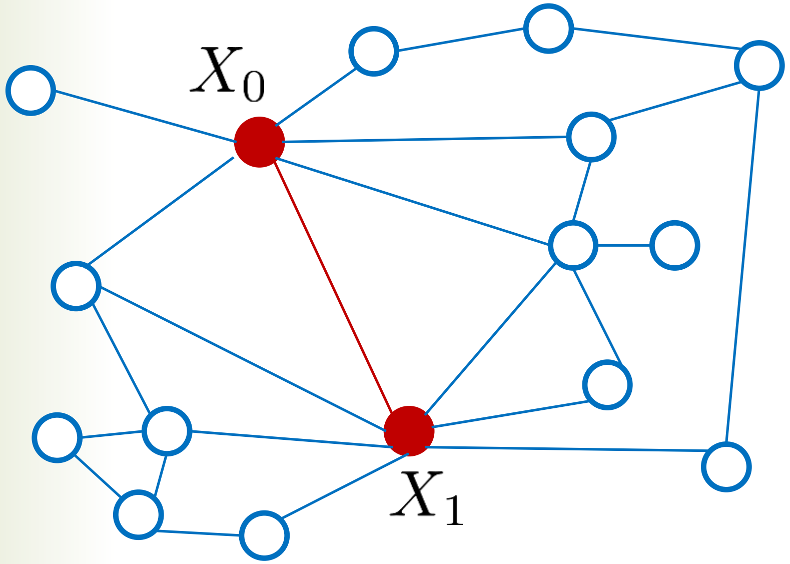

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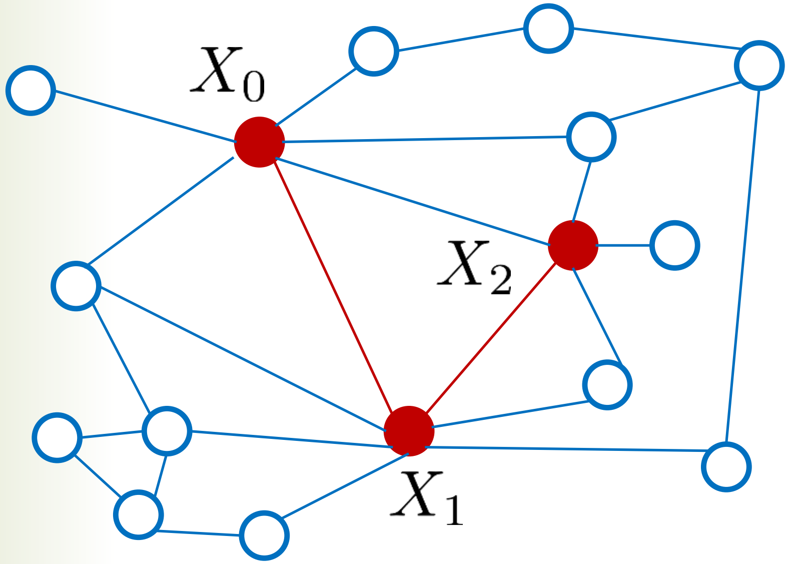

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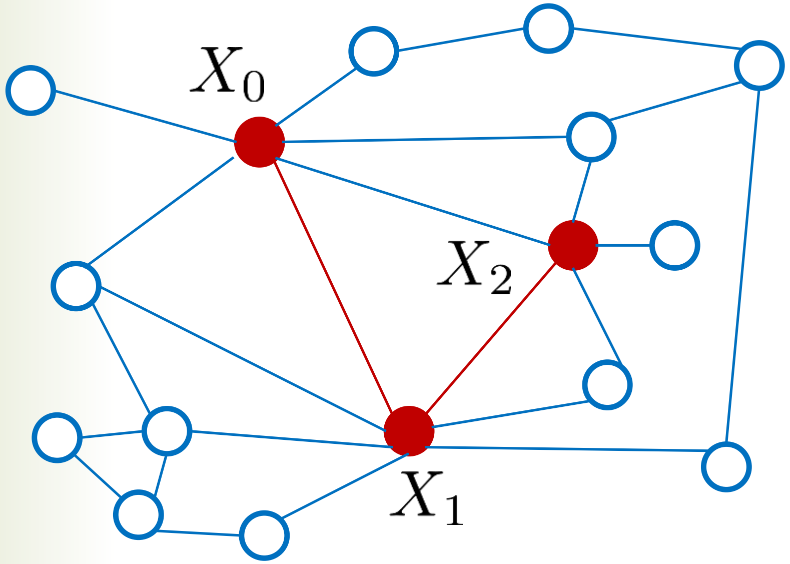

Random walk $\{X_k\}_{k\geq 0}$ has unique stationary distribution $\{\pi_i\}_{i=1}^{n}$ if graph G is connected and non-bipartite

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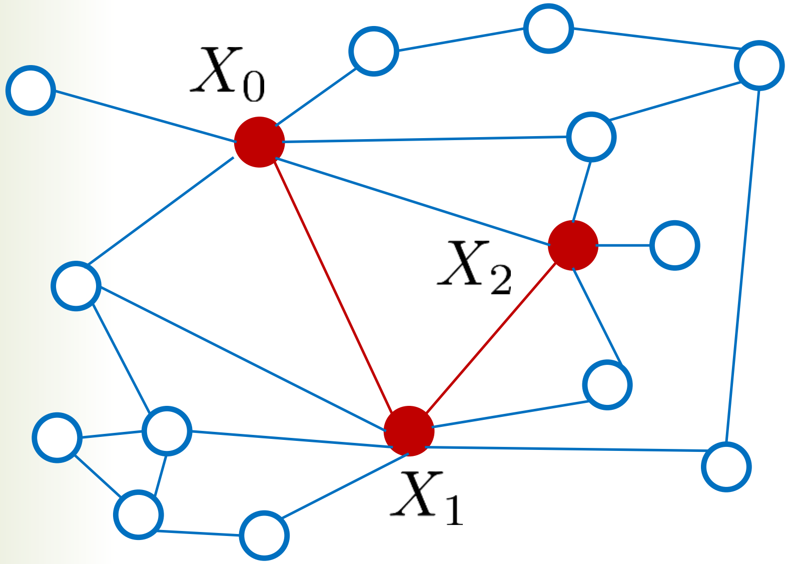

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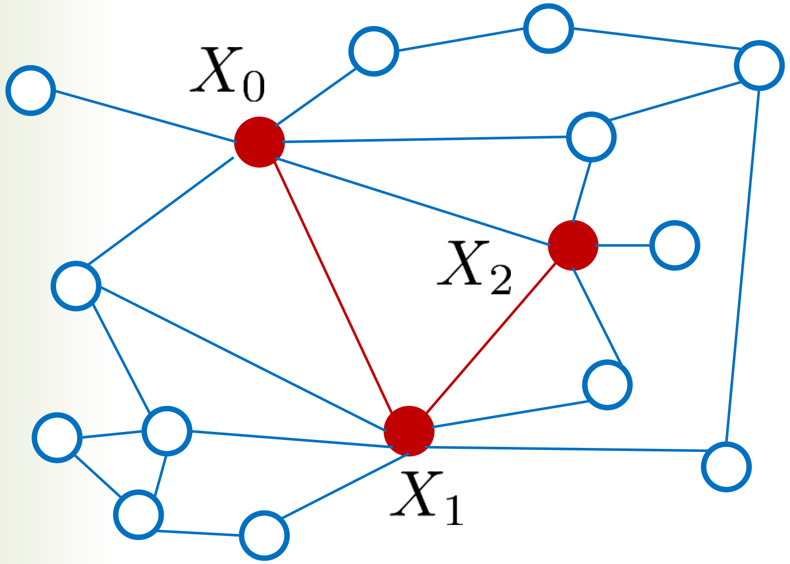

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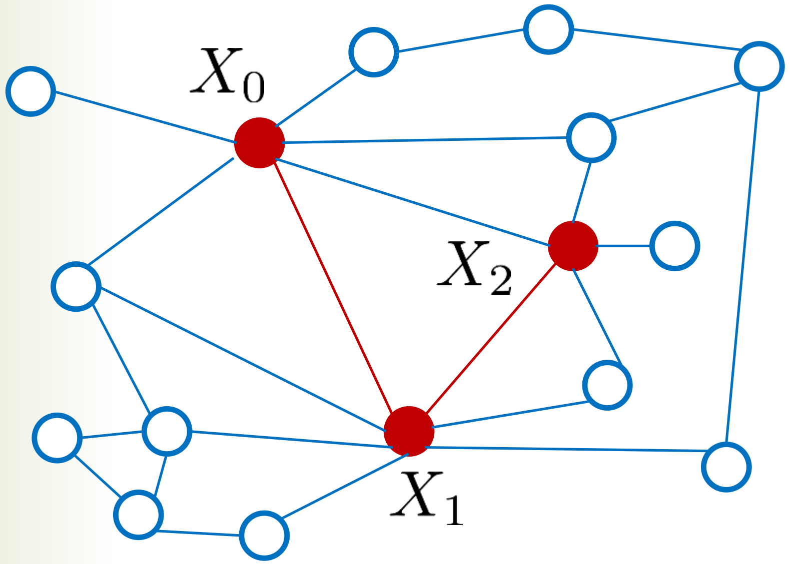

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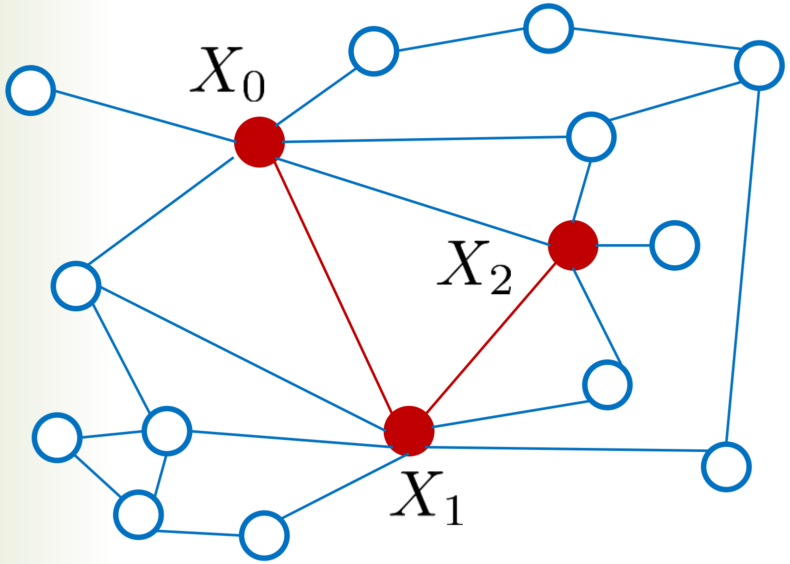

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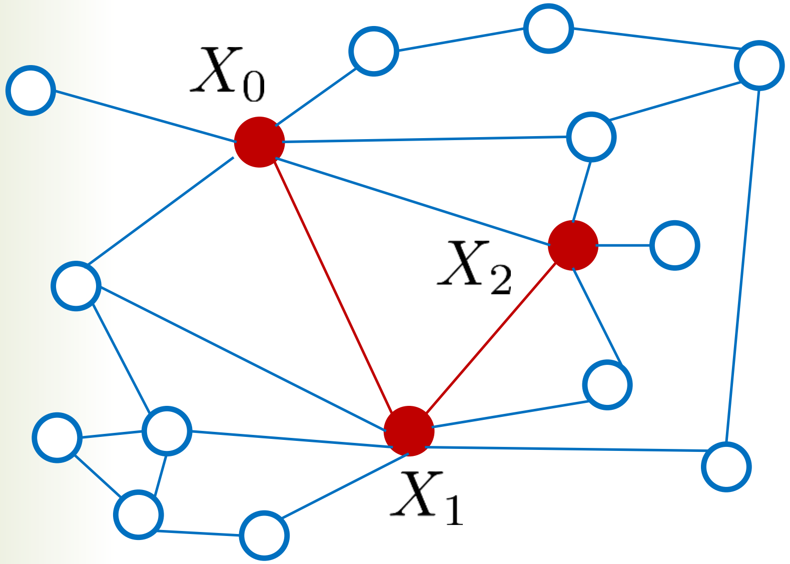

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How to make $\pi_u = \frac{1}{|\mathcal{V}|}$? How to compare different random walks?

Respondent Driven Sampling

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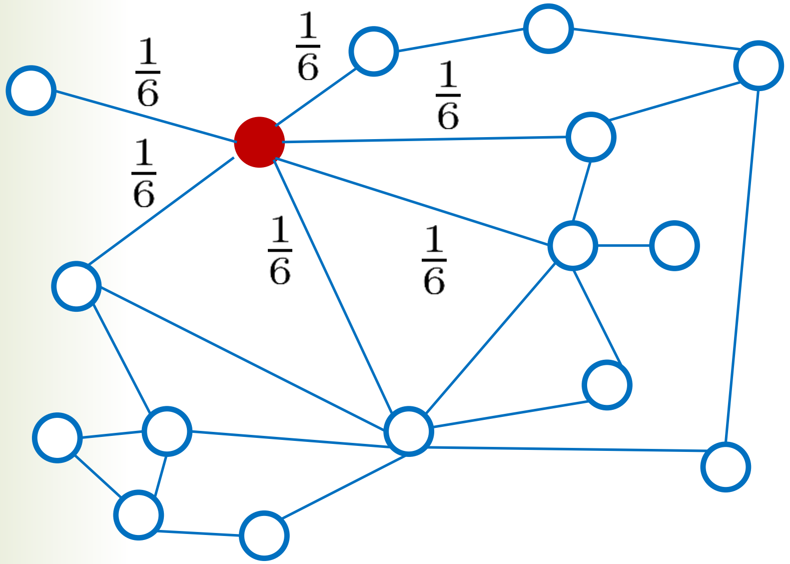

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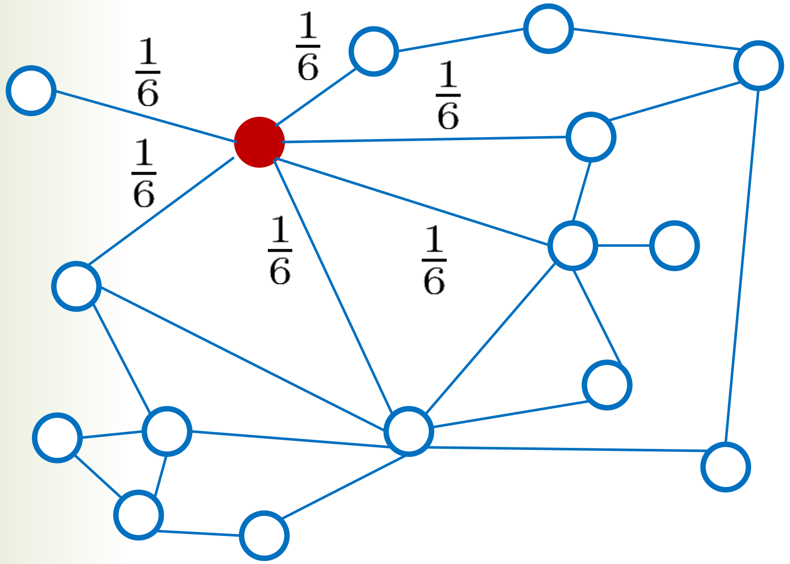

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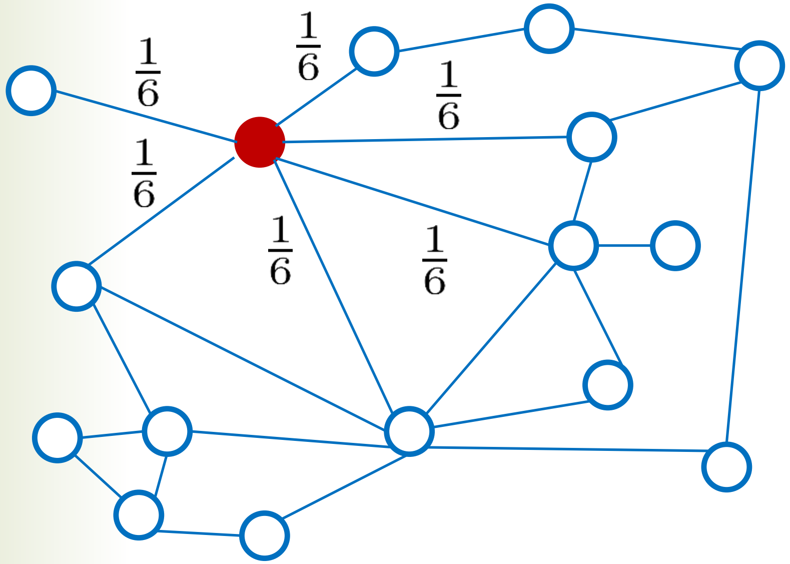

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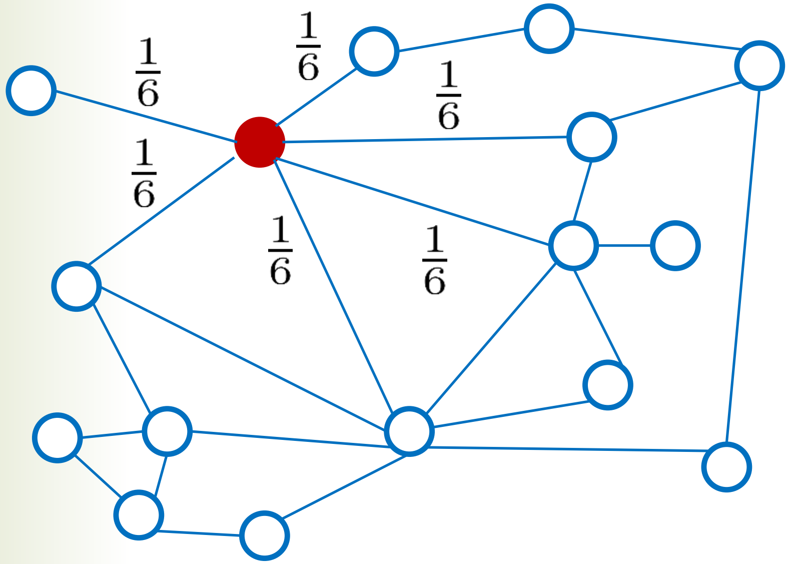

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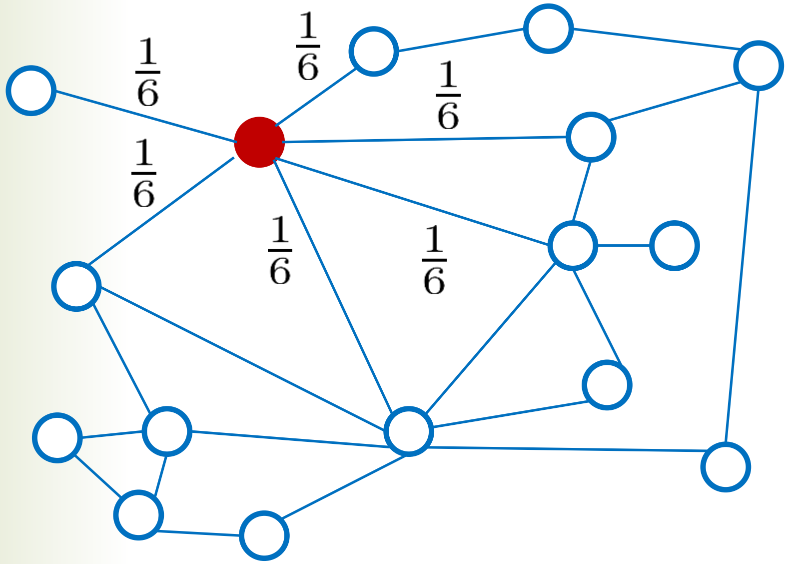

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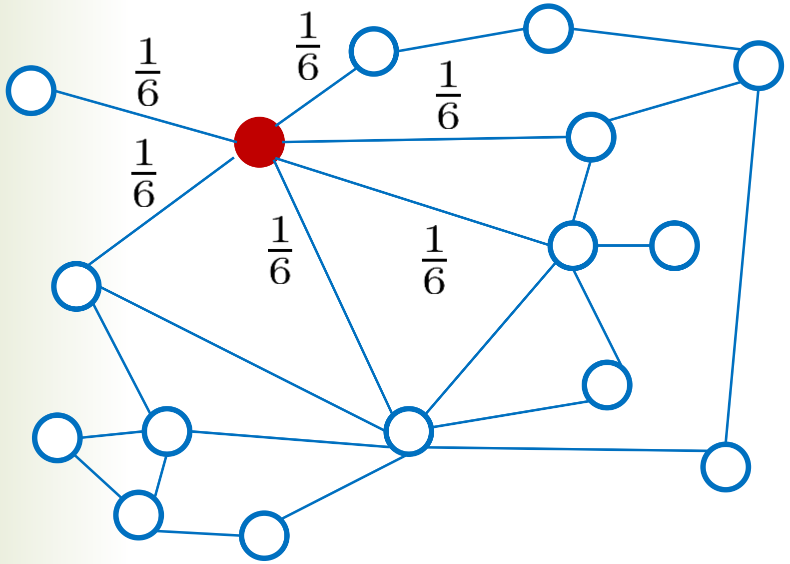

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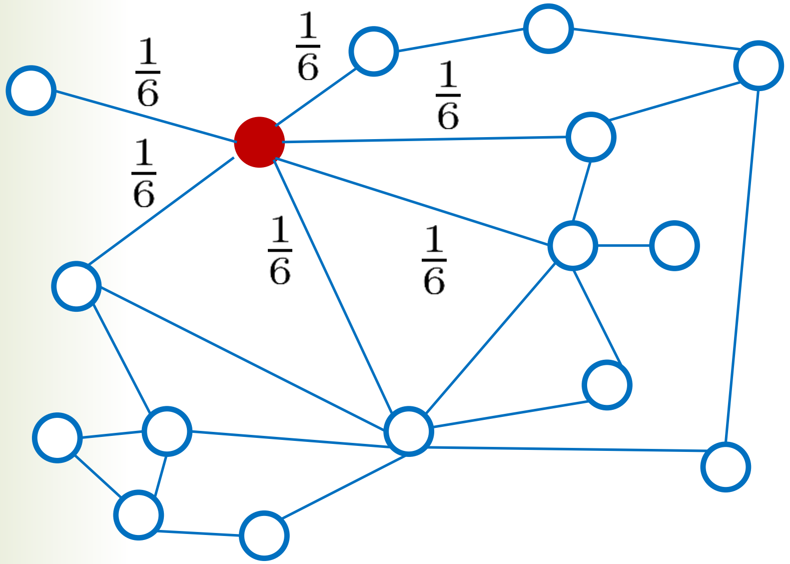

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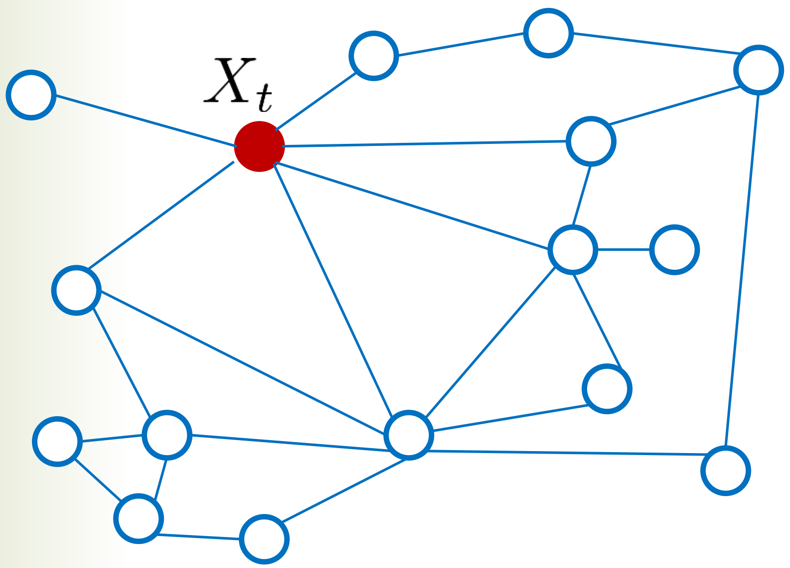
Metropolis Hastings Sampling



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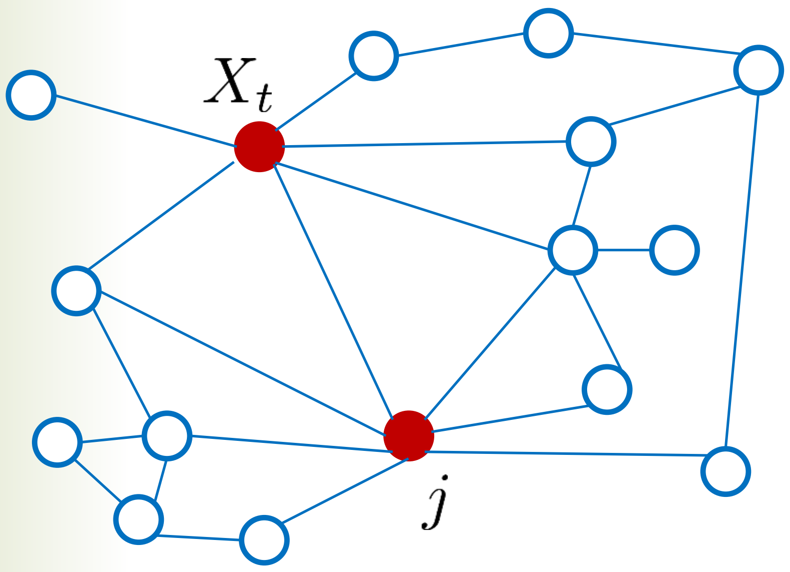

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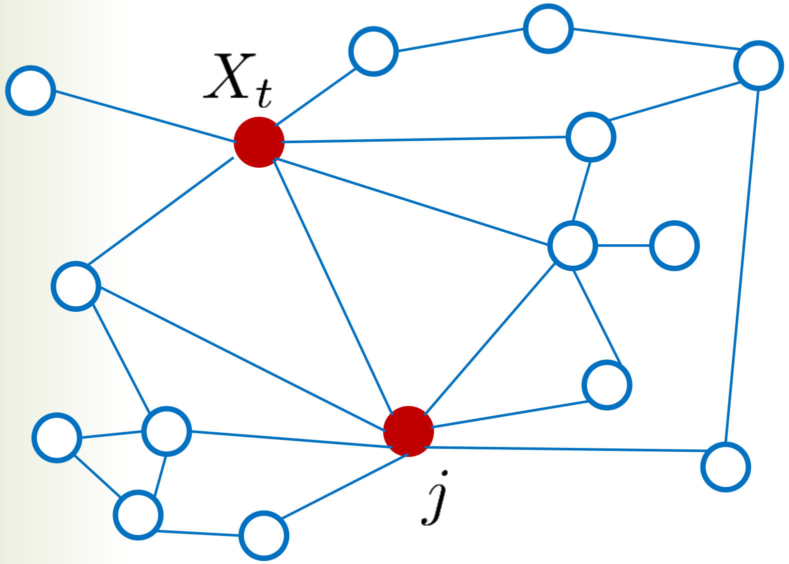

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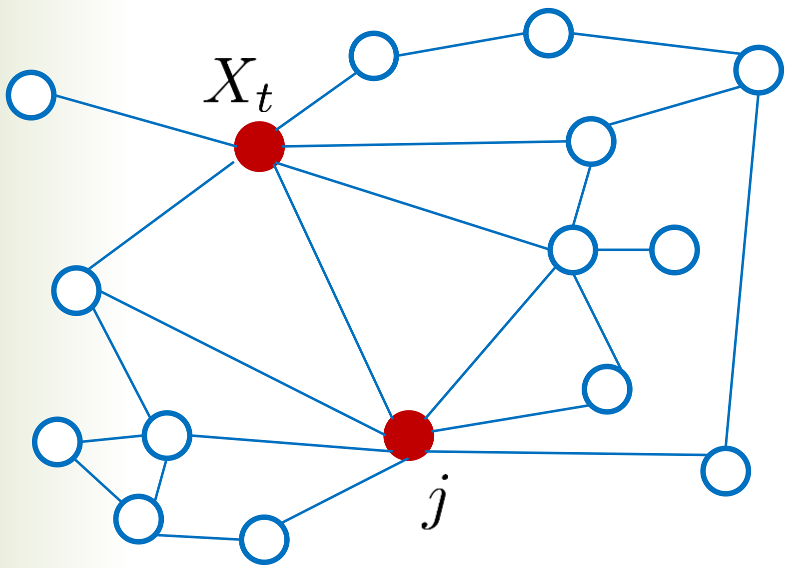

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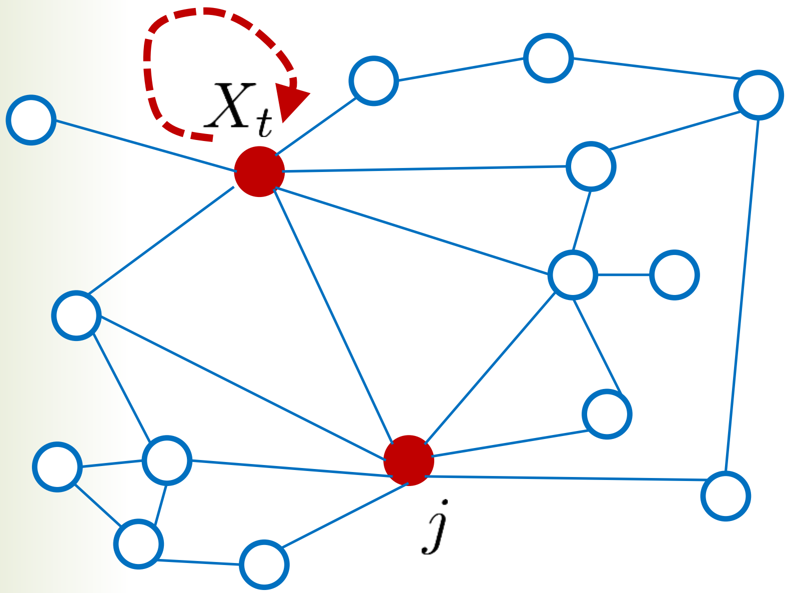

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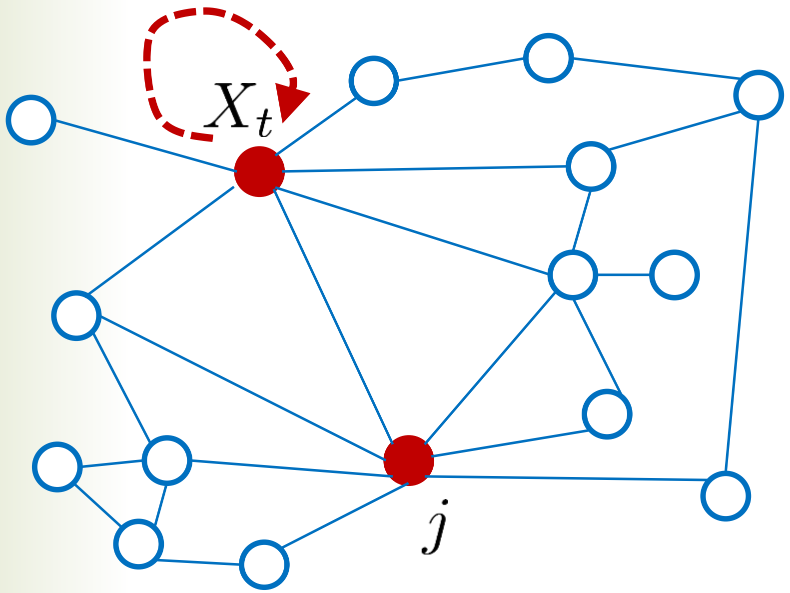

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- Graph not necessarily connected or
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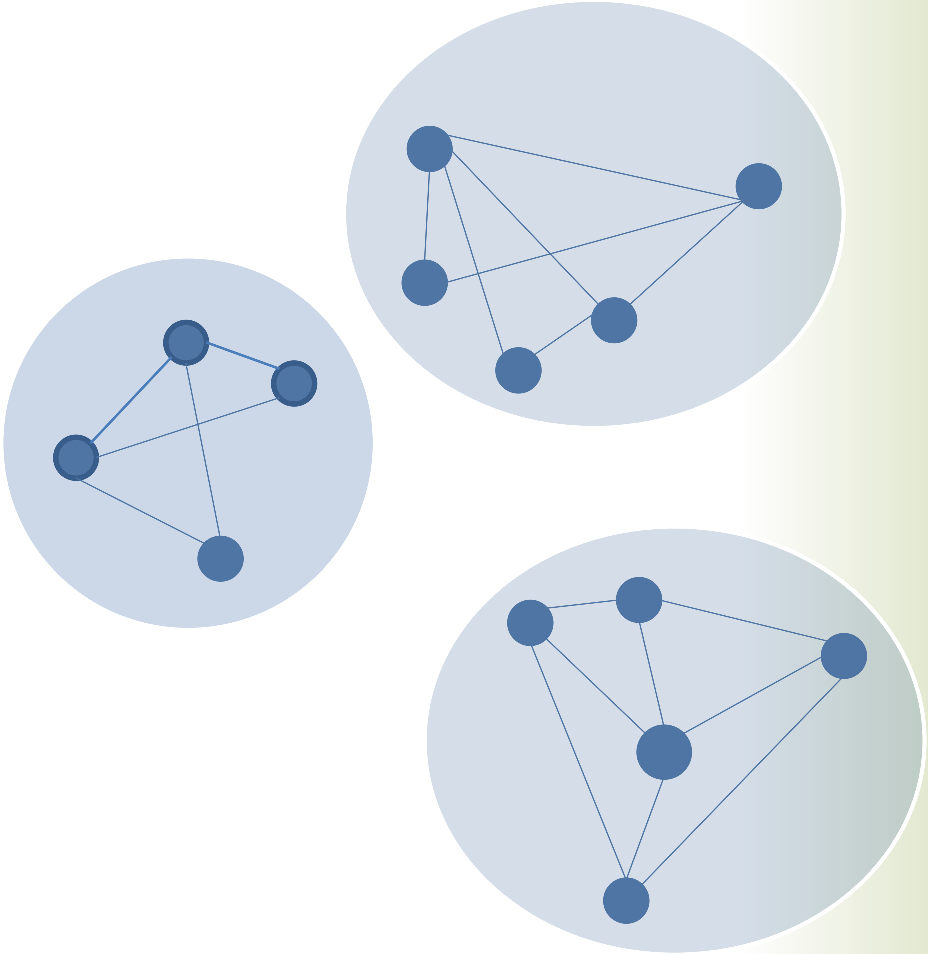

Reinforcement Learning technique

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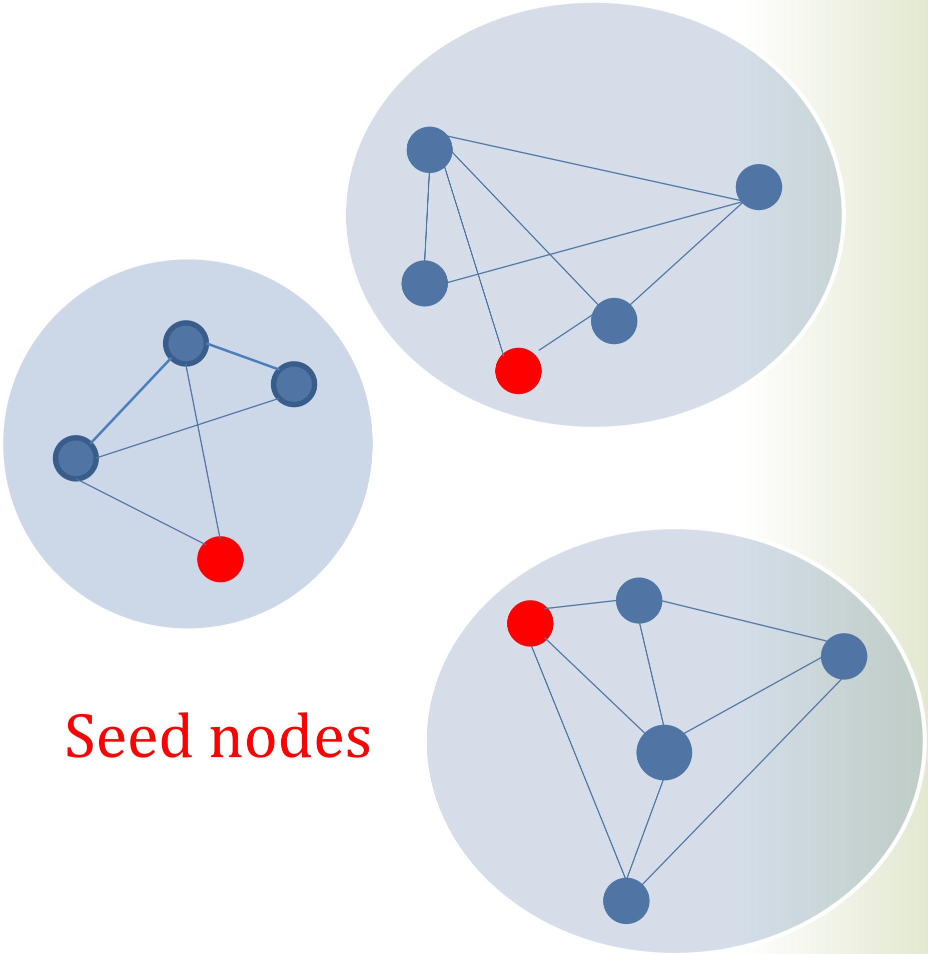
- Few seed nodes



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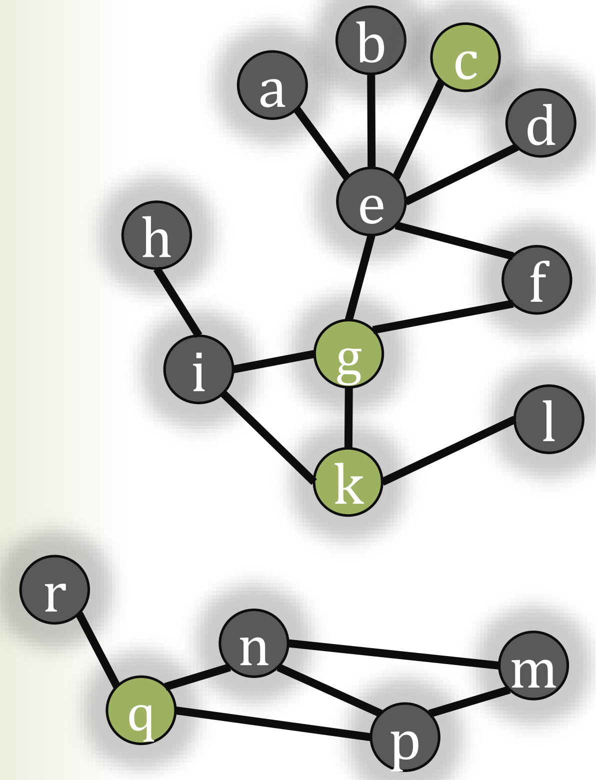

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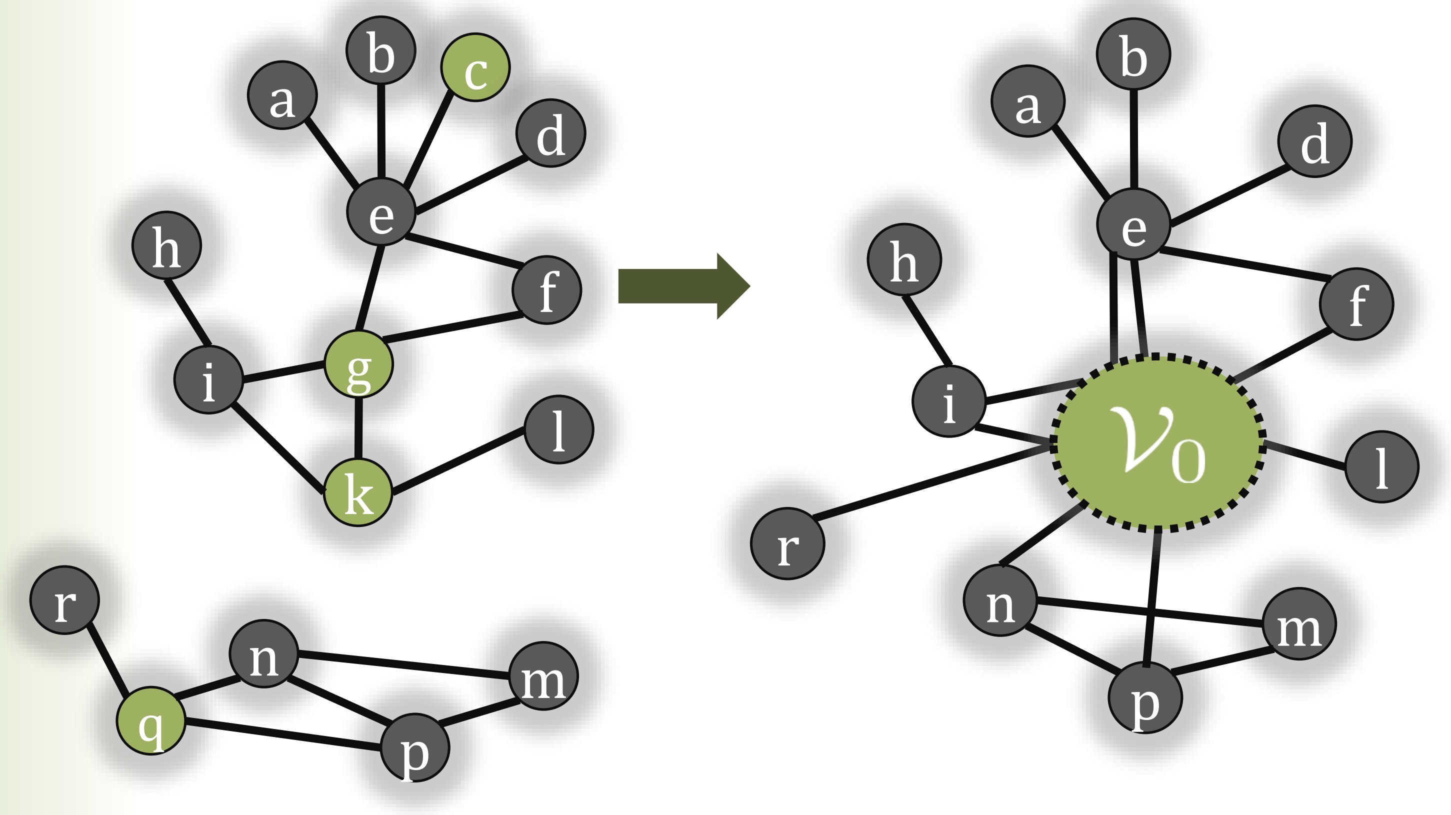

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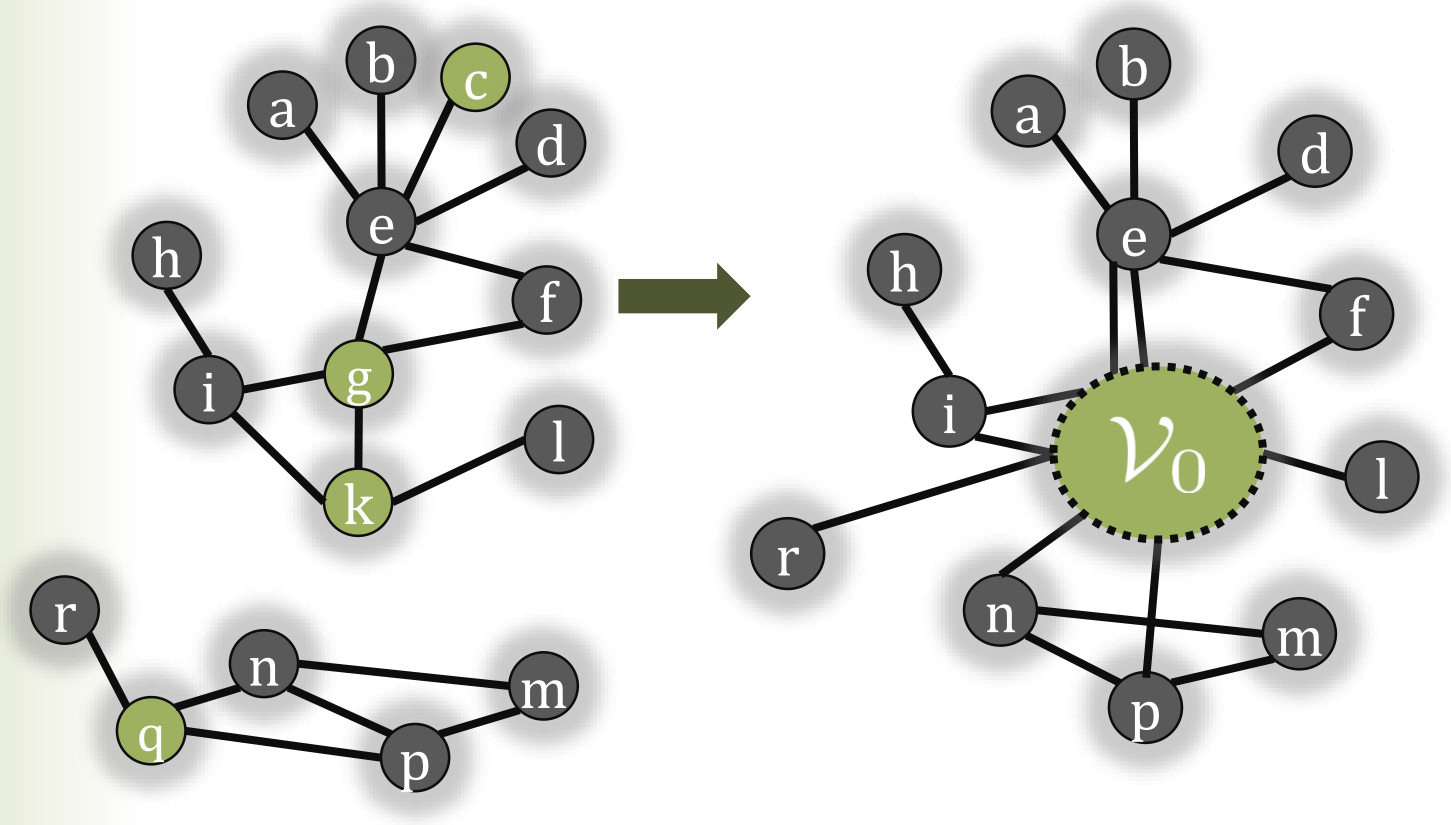

Sample
$z \sim \text{Uniform}(\mathcal{V}_0)$

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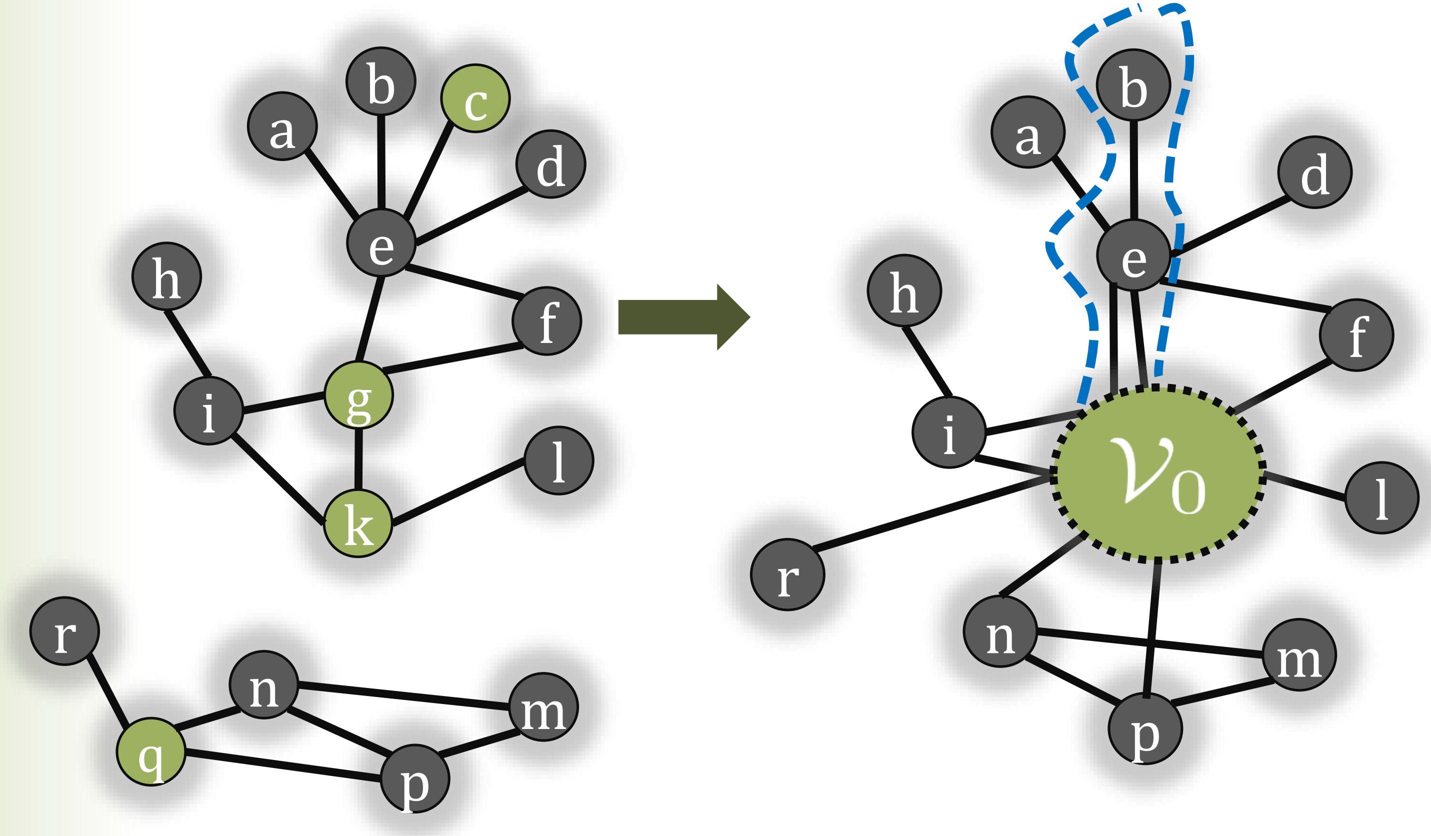

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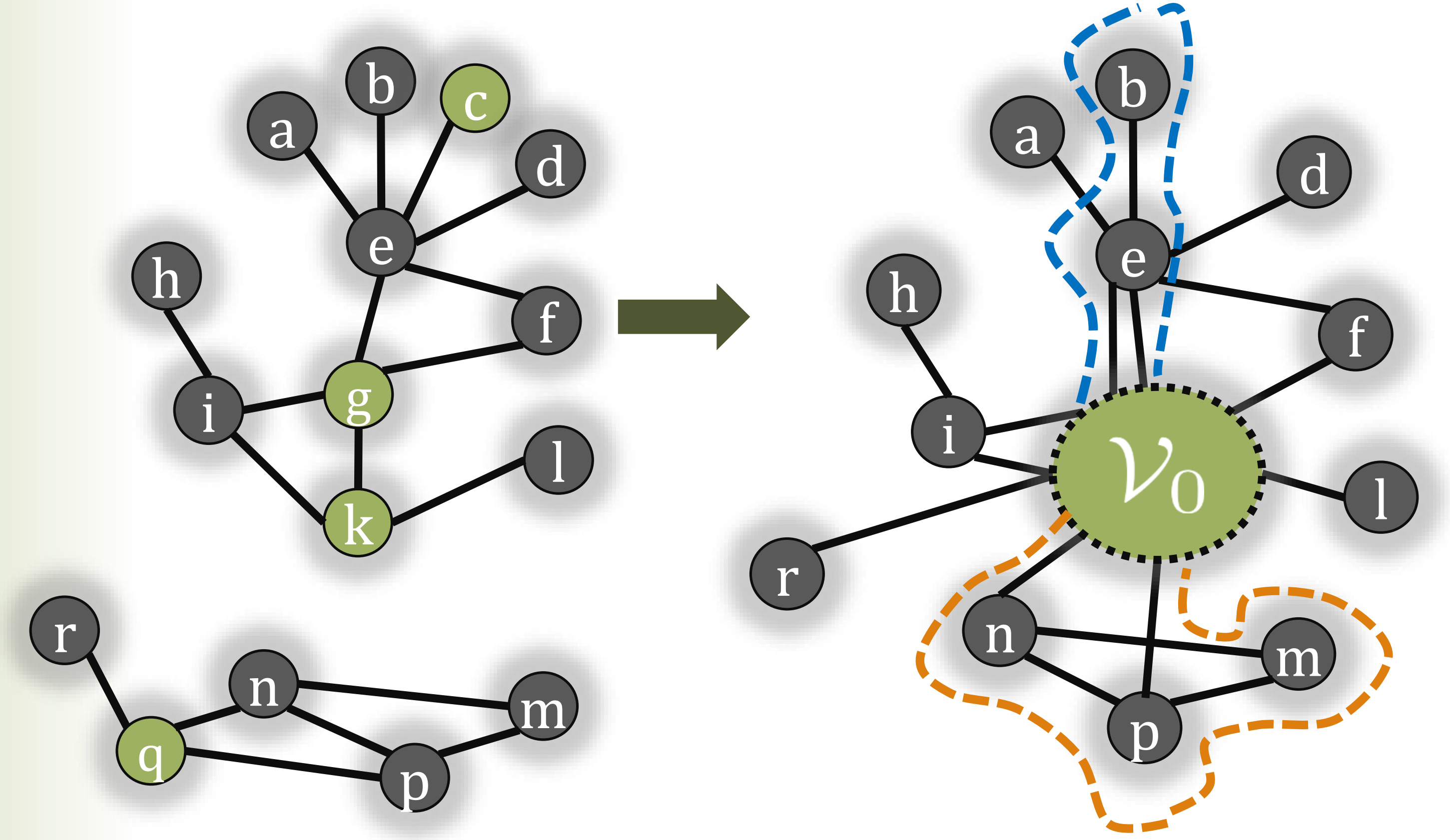

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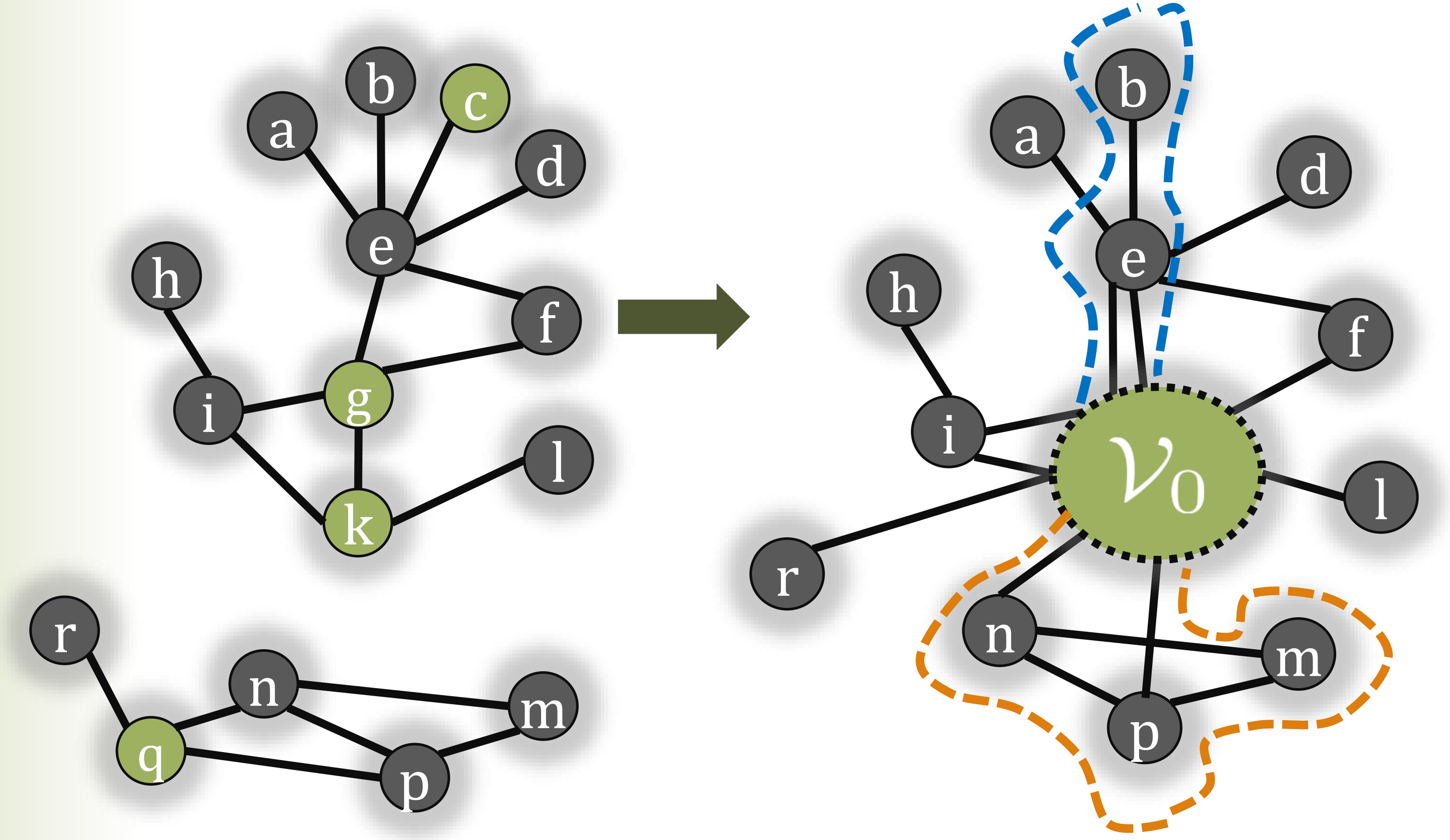

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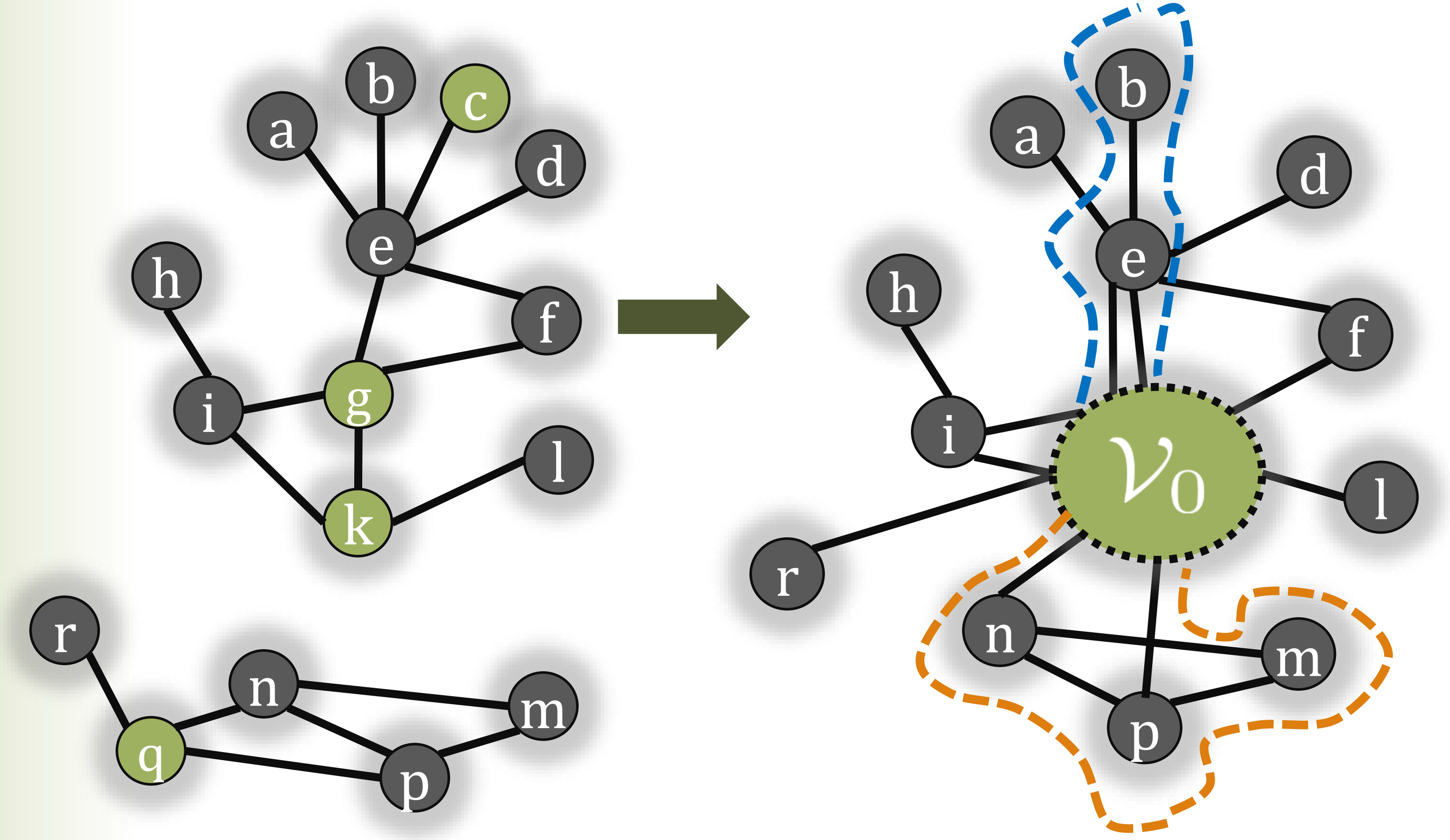

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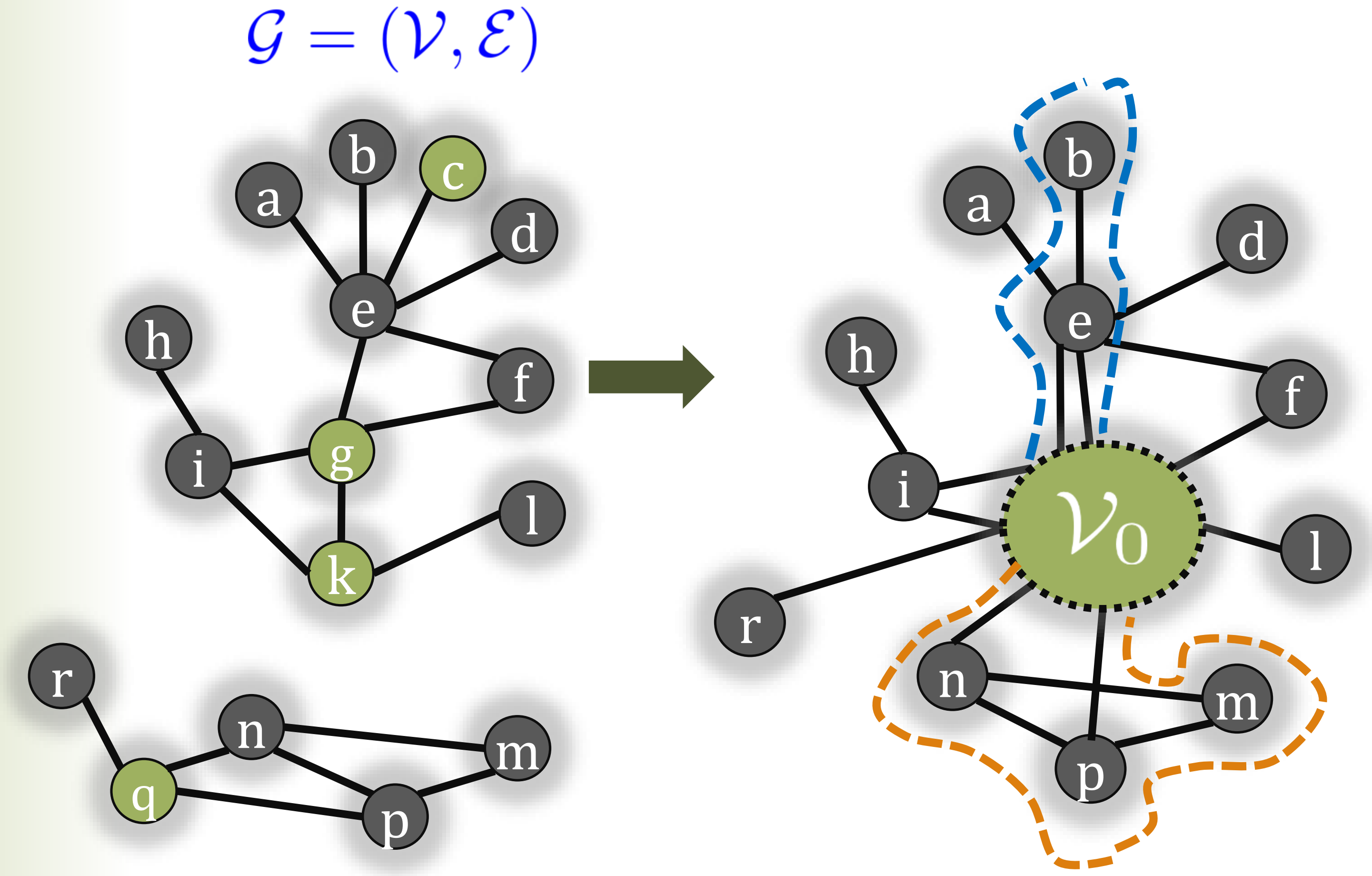

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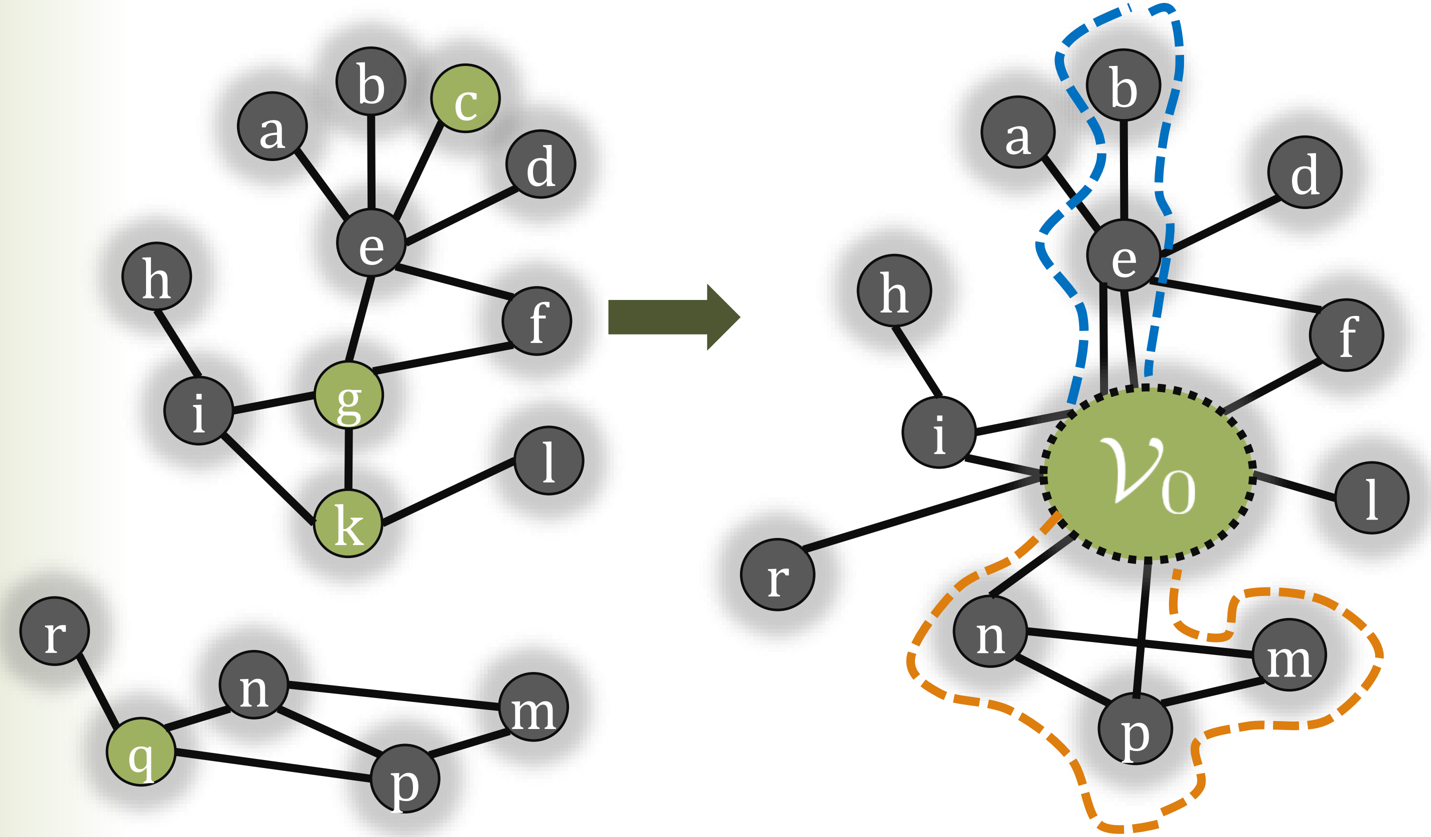

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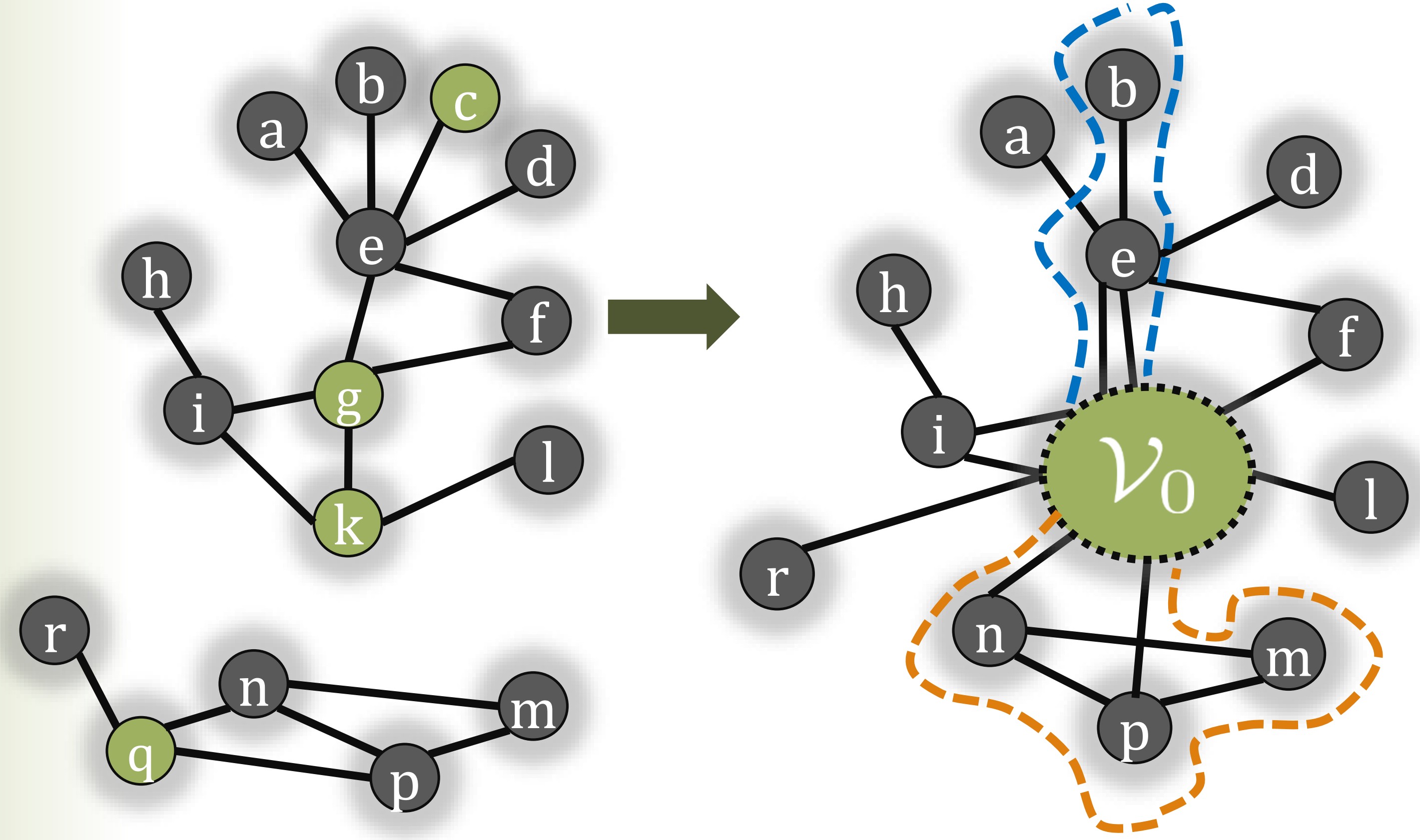

Properties of tours:

- Tours are independent
- Fully distributed crawler implementation
- Larger super node size, shorter the tours

Sample

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Details in the paper

Reinforcement Learning technique (contd.)

Stochastic Approximation Algorithm

Details in the paper

Reinforcement Learning technique (contd.)

For each node i in $\mathcal{V}_0$

Seed set

Stochastic Approximation Algorithm

Details in the paper

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Stochastic Approximation Algorithm

For each node i in $\mathcal{V}_0$ ← Seed set

$$V_{n+1}(i) = V_n(i) + a(n)\mathbb{I}\{z = i\} \left[\underbrace{\left(\sum_{m=1}^{\xi(n)} f(X_m^n) \right)}_{\text{Function sum inside a tour}} - \frac{\xi(n)}{|\mathcal{V}_0|} \sum_{j \in \mathcal{V}_0} V_n(j) + V_n(X_{\xi(n)}^n) - V_n(i) \right]$$

Cost function

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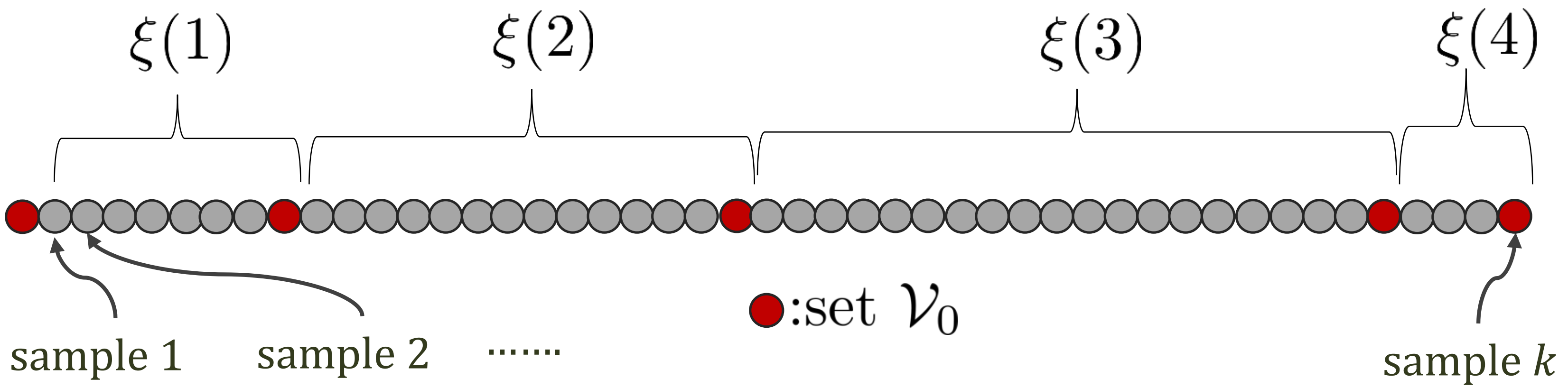

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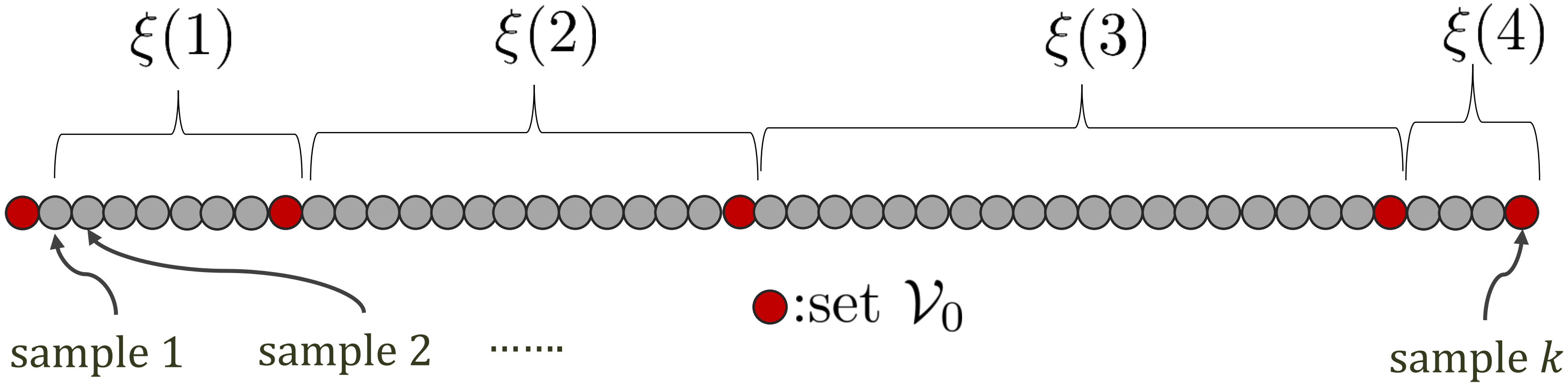
$a(n) > 0$ are stepsizes satisfying $\sum_n a(n) = \infty,\ \sum_n a(n)^2 < \infty.$



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Cost function ↑ ($V_{n+1}(i)$)

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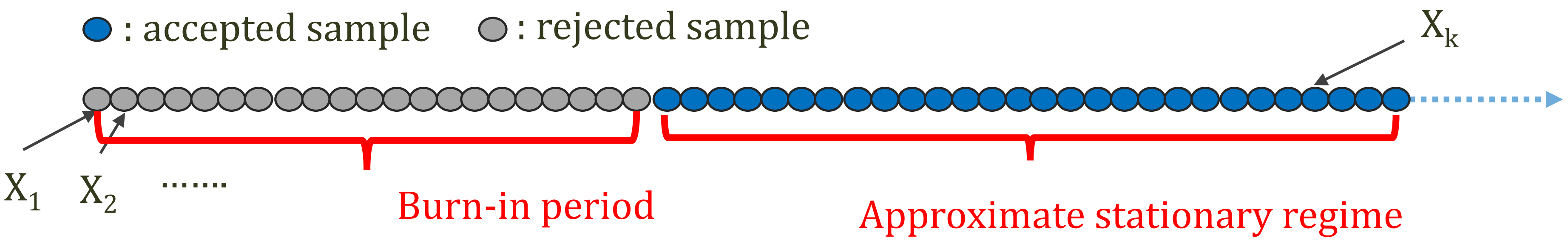

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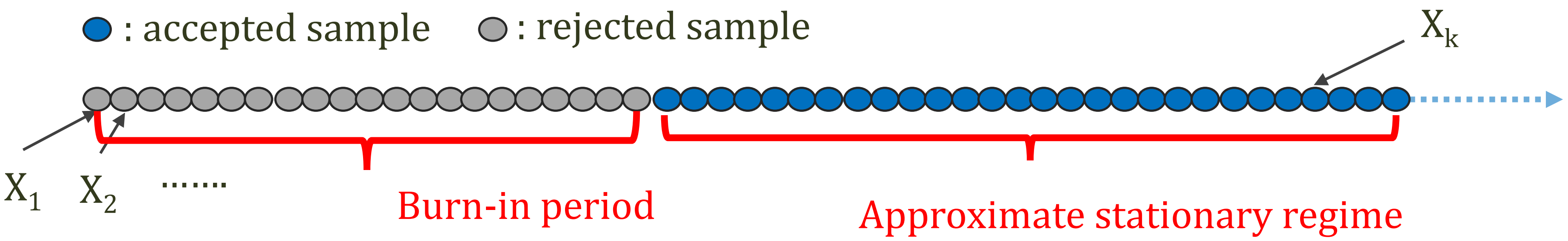

Reinforcement Learning technique does not require burn-in period

Which Random Walk method to select ?

- Mixing time

Not a good criterion here due to burn-in period.

- Efficiency of the estimator:

How many samples are needed to achieve certain accuracy

Asymptotic Variance

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Asymptotic variance of the estimator

$$\sigma^2 \triangleq \lim_{n\to\infty} n \,\mathrm{Var}\left(\mu^{(n)}(\mathcal{G})\right)$$

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Also from Central Limit Theorem equivalent

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Details in the paper

Asymptotic Variance (contd.)

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Asymptotic Variance (contd.)

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$$\text{where} \quad \sigma^2_{\mathrm{MH}} = \frac{2}{n}\mathbf{f}^T\mathbf{Z}\mathbf{f} - \frac{1}{n}\mathbf{f}^T\mathbf{f} - \left(\frac{1}{n}\mathbf{f}^T\mathbf{1}\right)^2$$

Fundamental matrix of Markov chain

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Details in the paper

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- For Reinforcement Learning based sampling,

$$\mathbb{E}\left[|\hat{\mu}^{(n)}_{\mathrm{RL}}(\mathcal{G}) - \mu(\mathcal{G})|^2\right] = \mathcal{O}\left(\frac{1}{n}\right)$$

Numerical Studies

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Normalized Root Mean Square Error (NRMSE) vs Budget B

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Budget B: number of allowed samples

Les Misérables network

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Number of nodes: 77, number of edges: 254.

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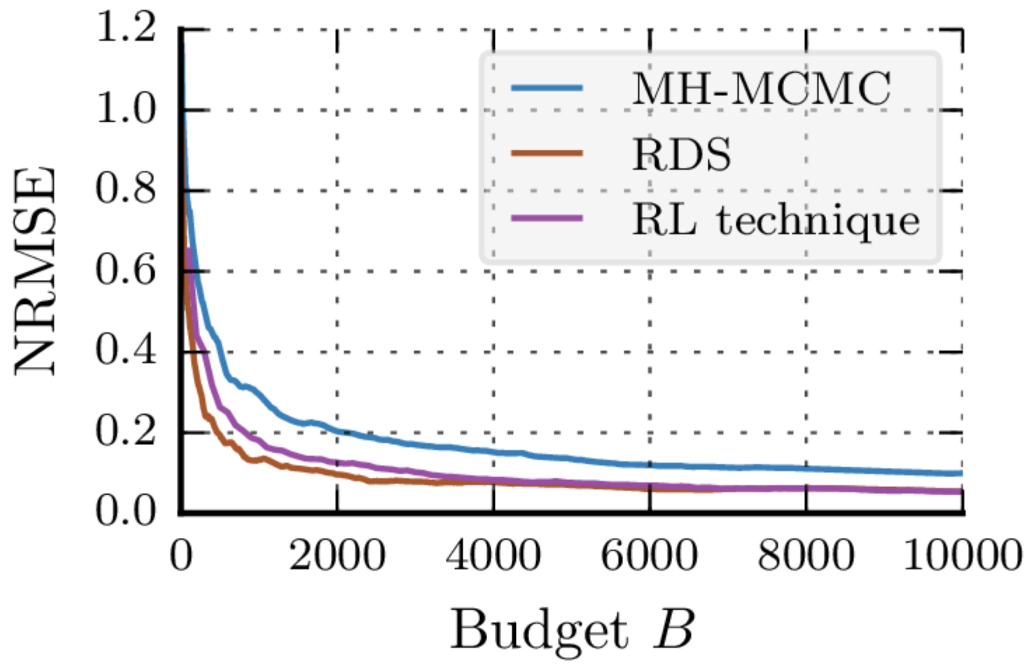

$$\mu(\mathcal{G}) = \frac{1}{|\mathcal{V}|} \sum_{u \in \mathcal{V}} \mathbb{I}\{\deg(\mathrm{u}) > 10\}$$

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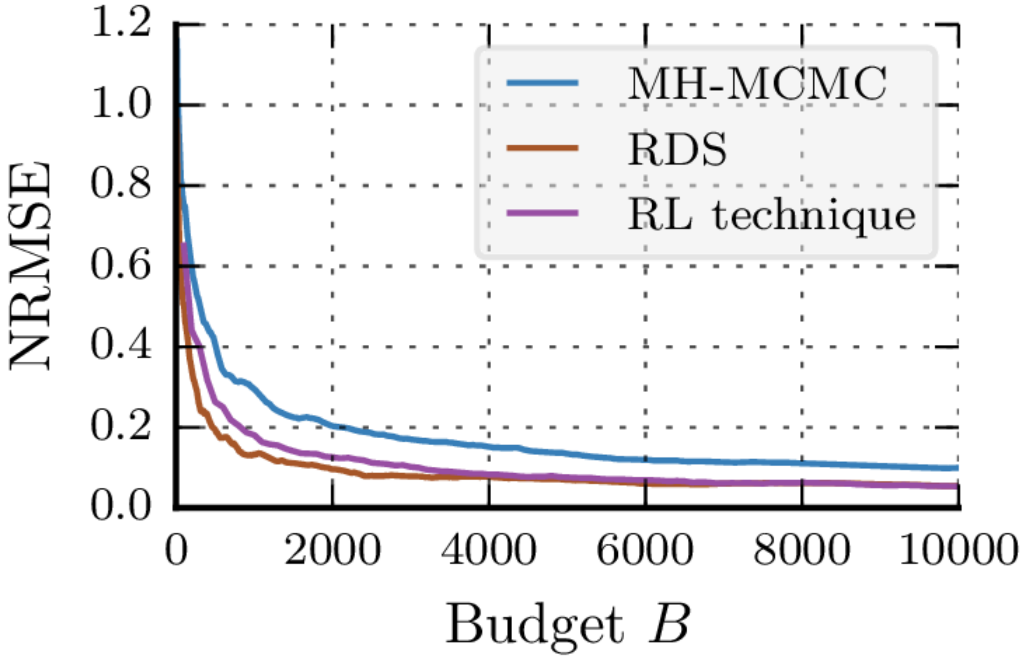

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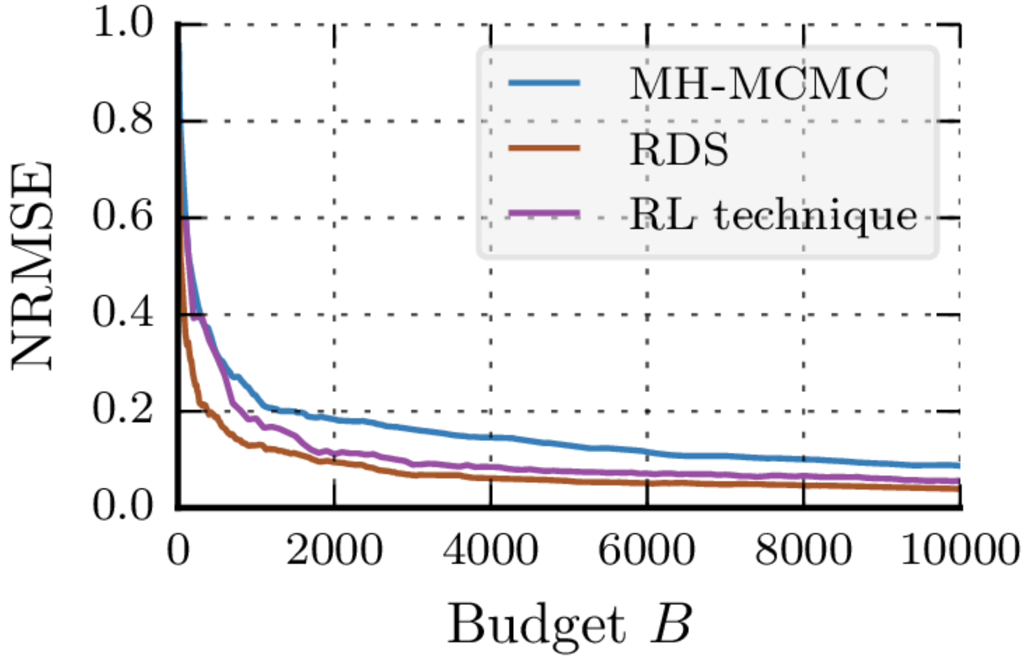

$$\mu(\mathcal{G}) = \frac{1}{|\mathcal{V}|} \sum_{u \in \mathcal{V}} \mathbb{I}\{\mathrm{deg(u)} < 4\}$$

Les Misérables network contd.

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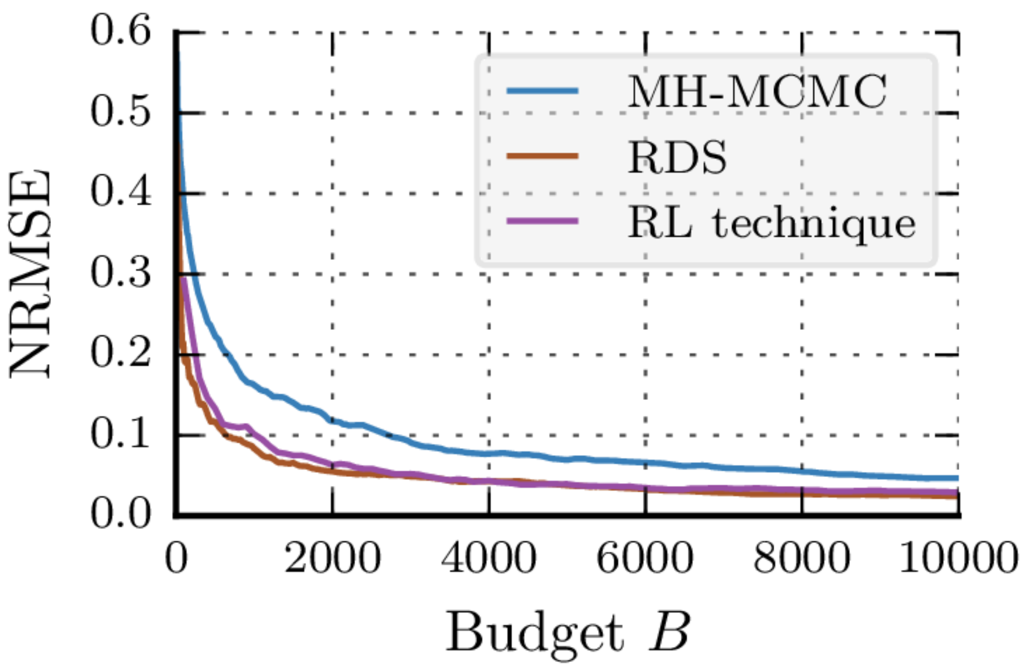

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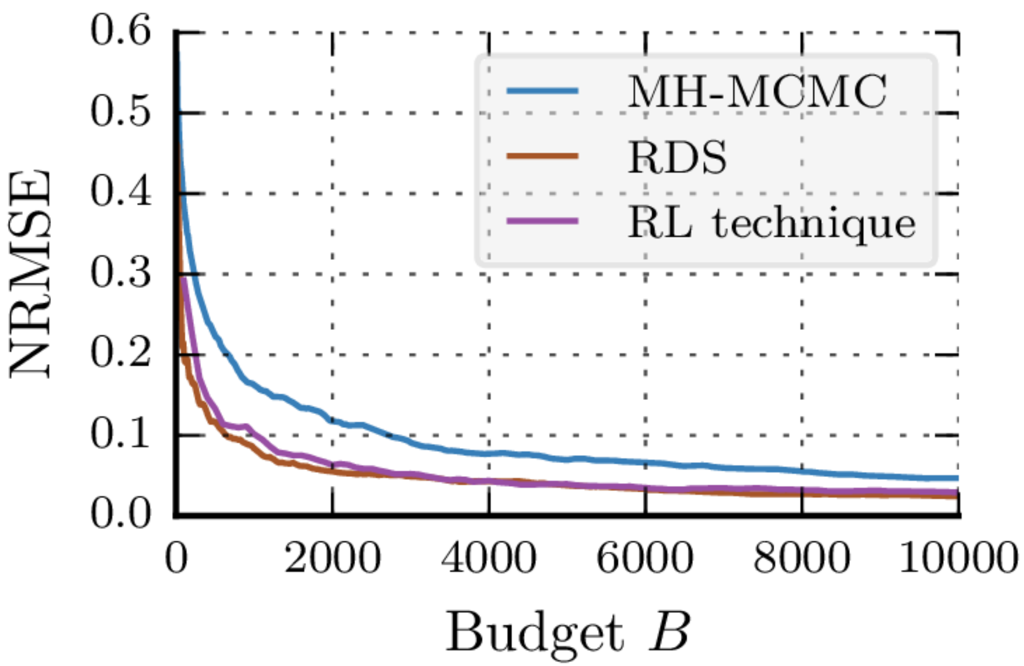
Les Misérables network contd.



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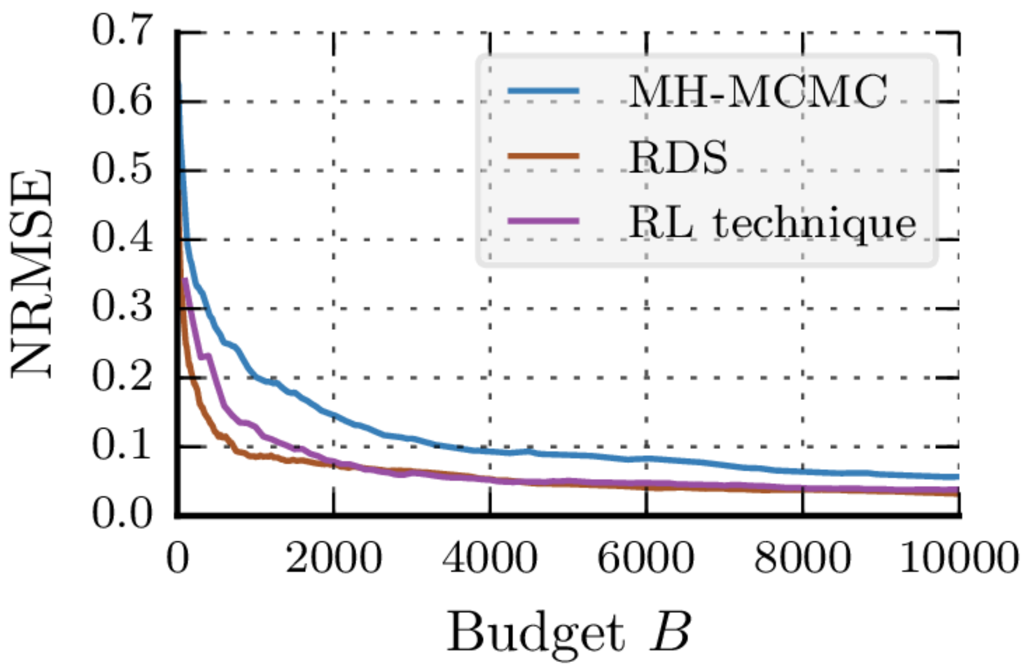

$\mu(\mathcal{G})$ = Average clustering coefficient

Les Misérables network contd.

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Study of asymptotic variance

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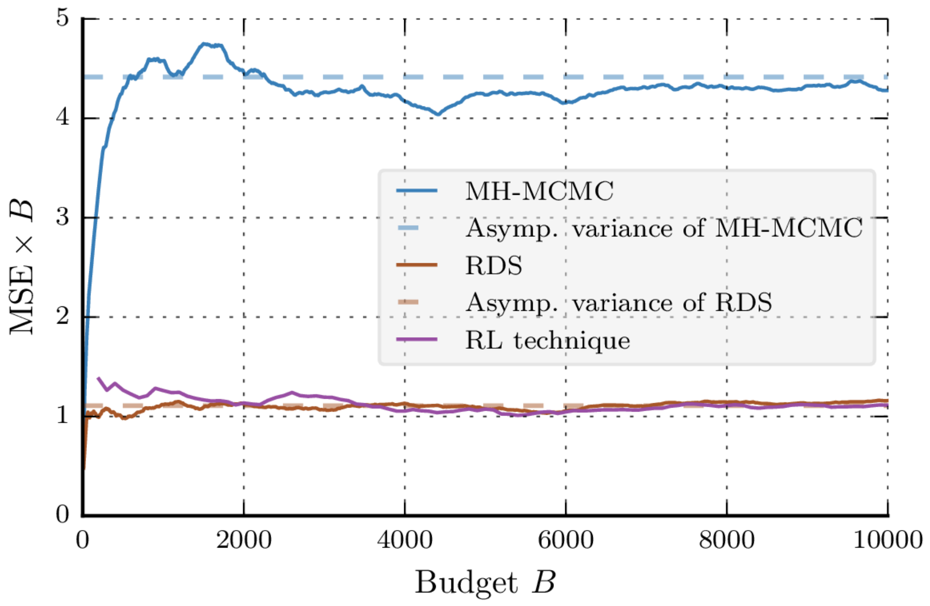
Study of asymptotic variance



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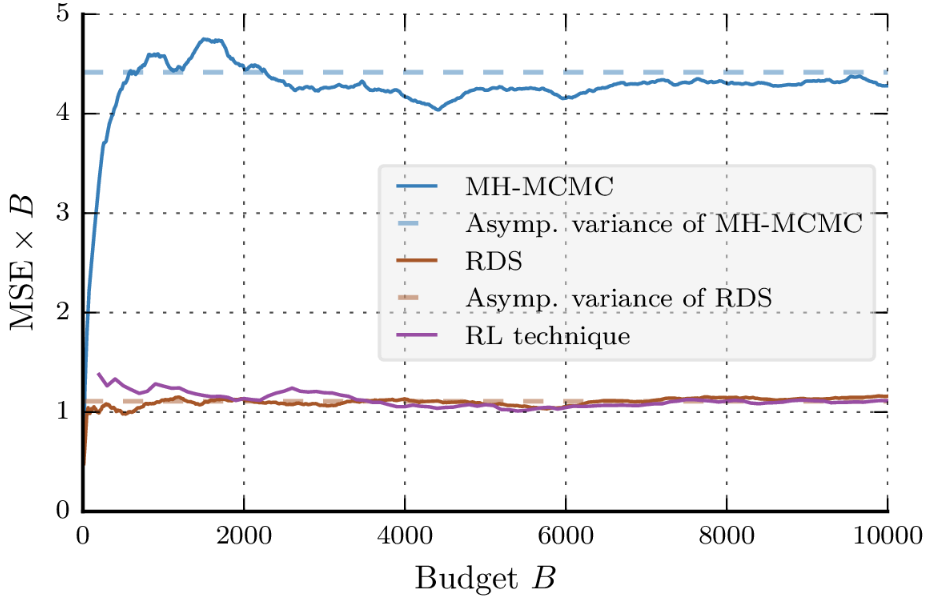
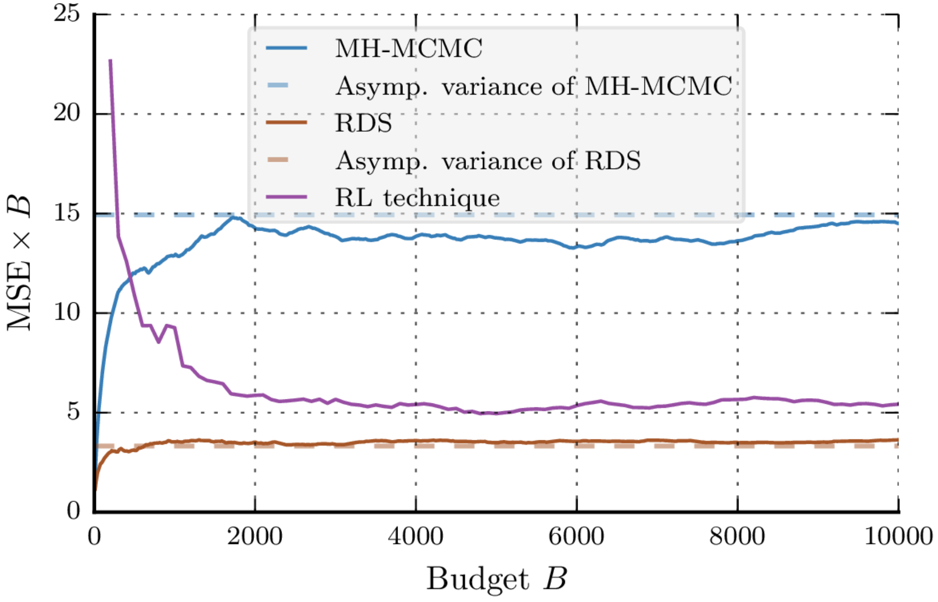
Les Misérables network contd.

Study of asymptotic variance





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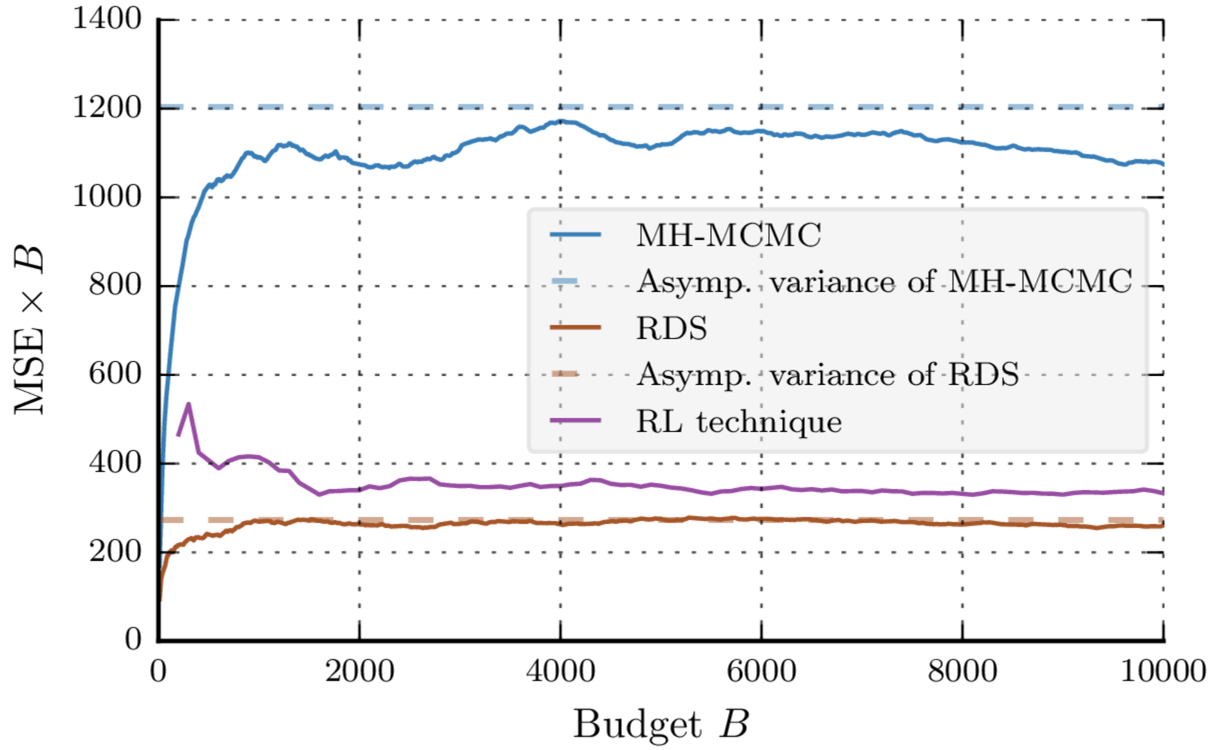

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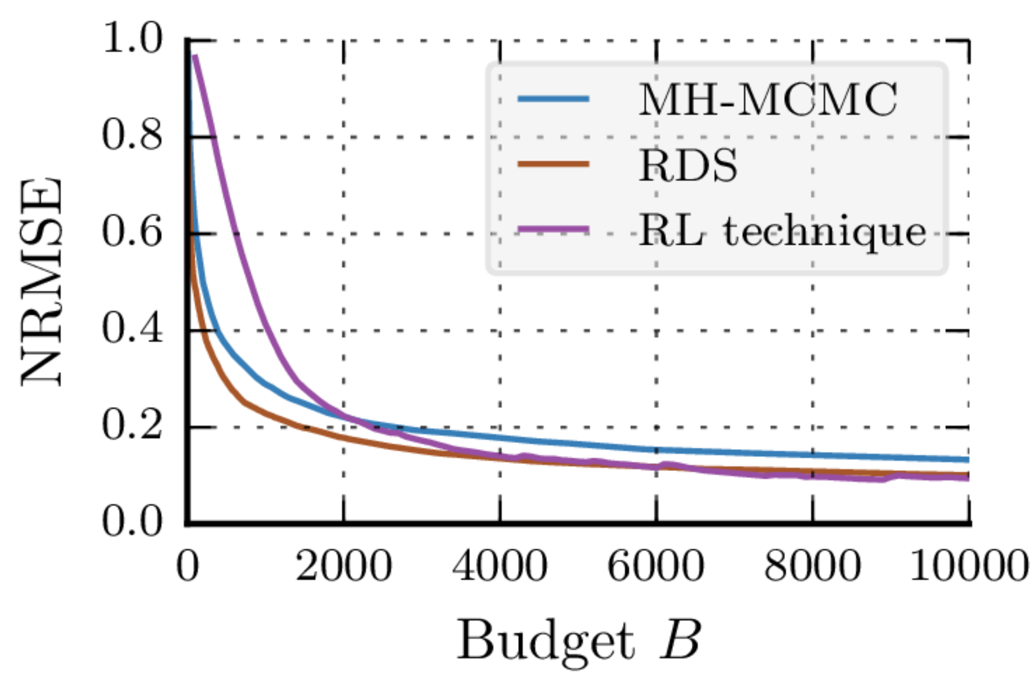

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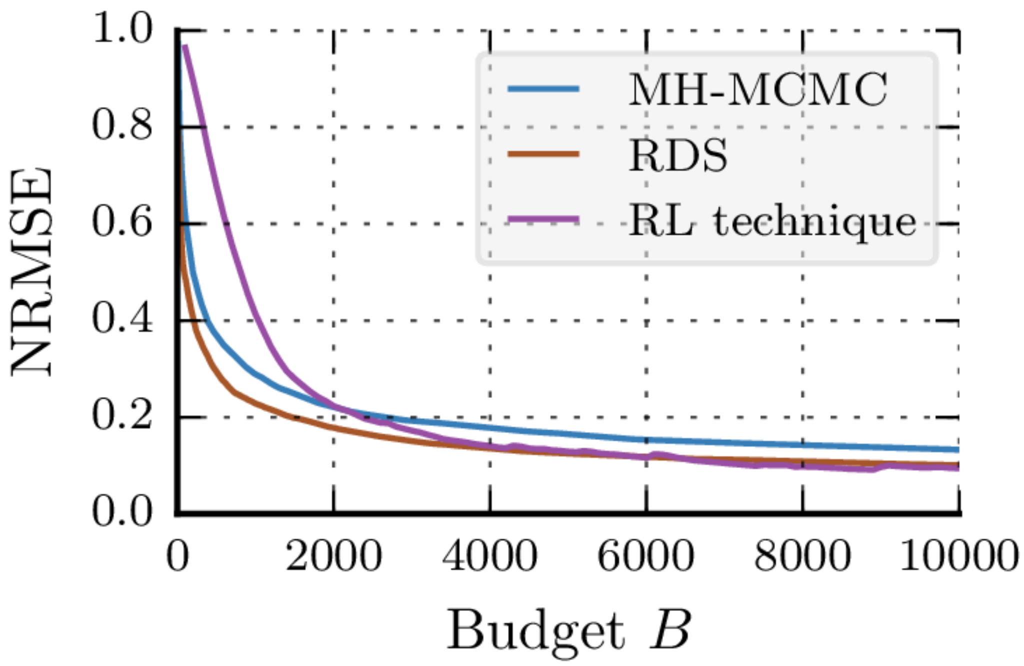

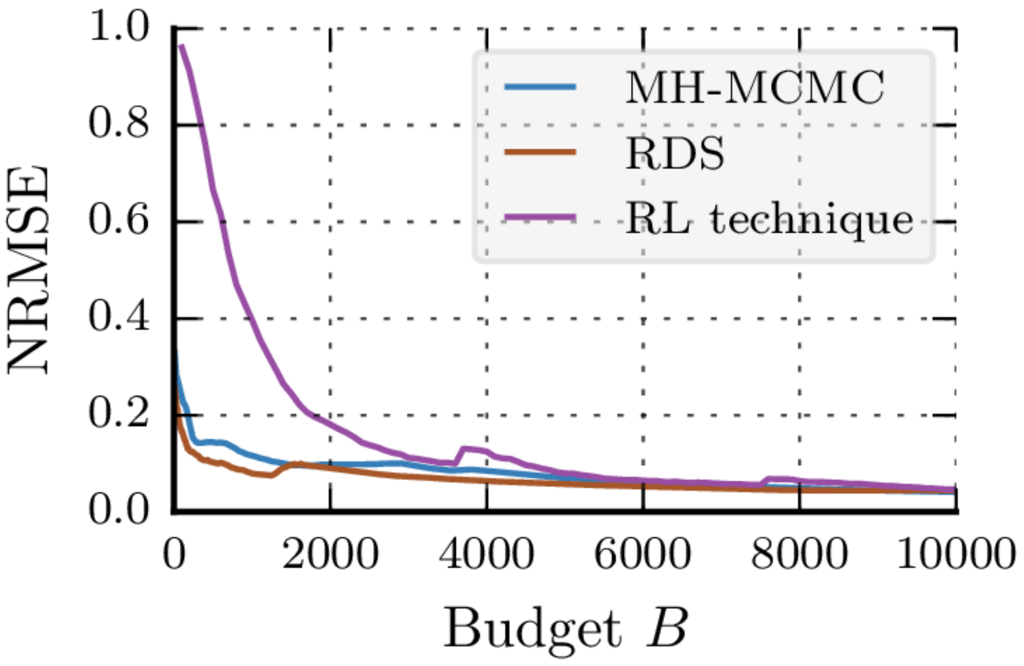

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Friendster network contd.

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Stability of sample paths:

Friendster network contd.

Stability of sample paths: single path example

Friendster network contd.

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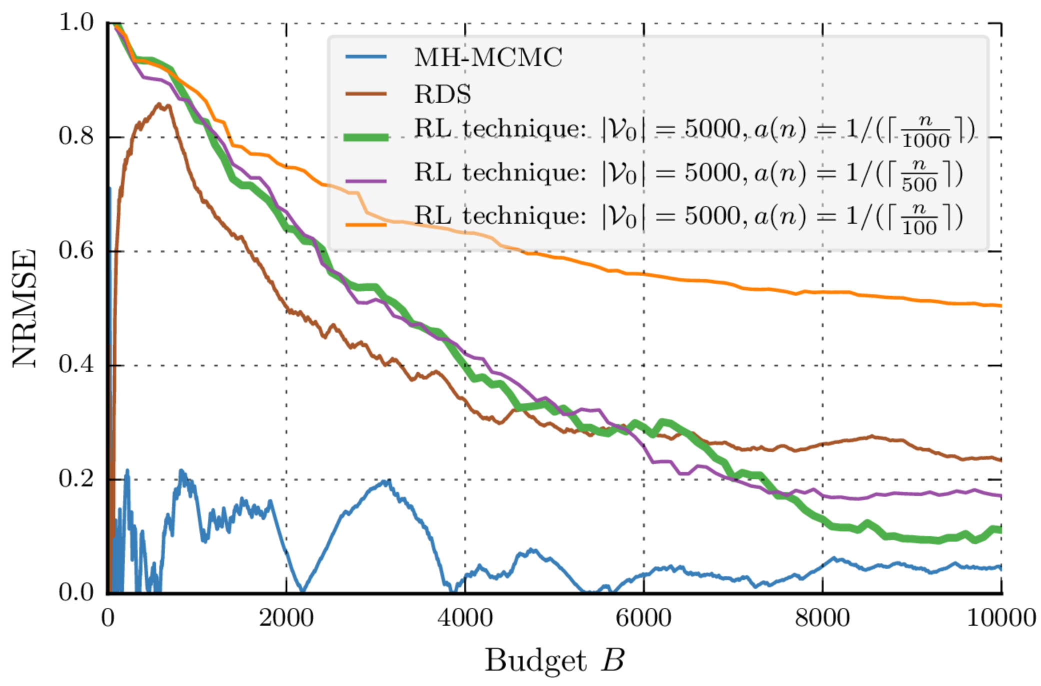
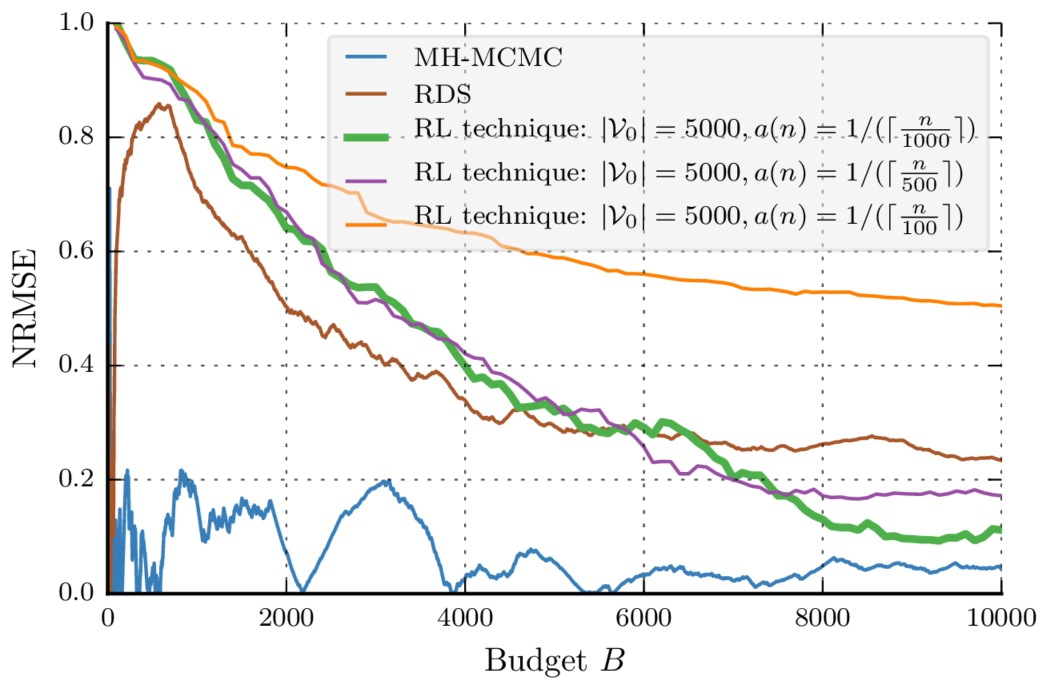
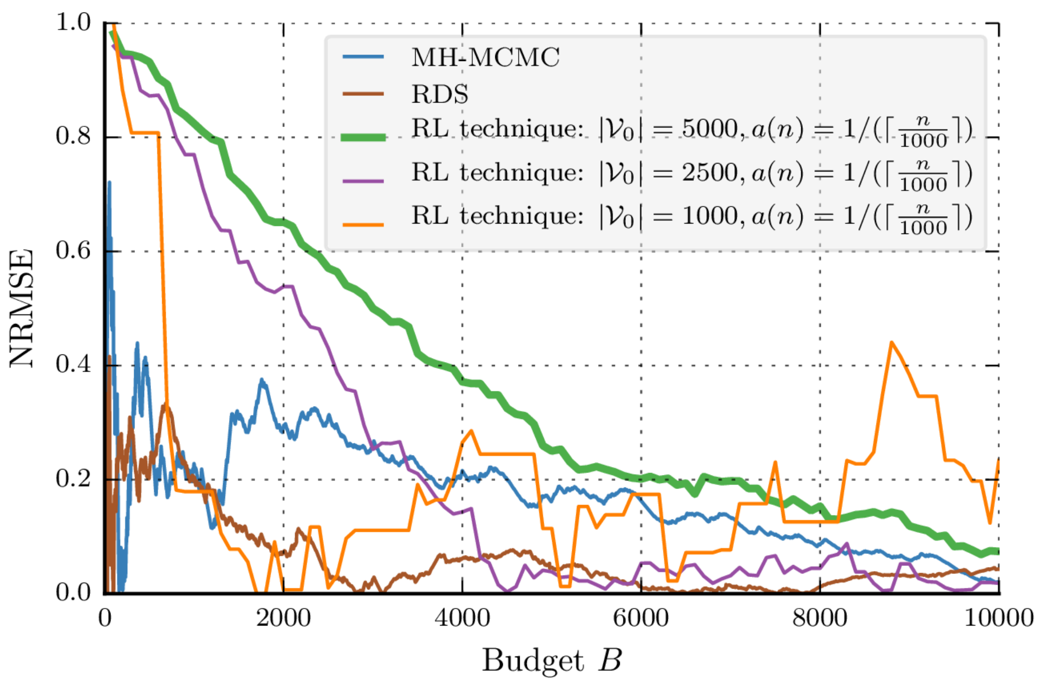


Varying super-node size

Friendster network contd.

Stability of sample paths: single path example





Varying super-node size

Varying step size

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- RDS works better. RL technique comparable, yet more stable and no burn-in !

Thank you!

http://bit.do/Jithin