A Novel Algorithm for Cooperative Distributed Sequential Spectrum Sensing in Cognitive Radio

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***Abstract*—This paper considers cooperative spectrum sensing in Cognitive Radios. In our previous work we have developed DualSPRT, a distributed algorithm for cooperative spectrum sensing using Sequential Probability Ratio Test (SPRT) at the Cognitive Radios as well as at the fusion center. This algorithm works well, but is not optimal. In this paper we propose an improved algorithm- SPRT-CSPRT, which is motivated from Cumulative Sum Procedures (CUSUM). We analyse it theoretically. We also modify this algorithm to handle uncertainties in SNR's and fading.**

Keywords- Cognitive Radio, Spectrum Sensing, Cooperative Distributed Algorithm, SPRT.

I. Introduction

Presently there is a scarcity of wireless spectrum worldwide due to an increase in wireless services. Cognitive Radios are proposed as a solution to this problem. They access the spectrum licensed to existing communication services (primary users) opportunistically and dynamically without causing much interference to the primary users. This is made possible via spectrum sensing by the Cognitive Radios (secondary users), to gain knowledge about the spectrum usage by the primary devices. However due to the strict spectrum sensing requirements [20] and the various wireless channel impairments spectrum sensing has become the main challenge faced by the Cognitive Radios.

Cooperative spectrum sensing ([**?**], [21], [24]) in which different Cognitive Radios communicate each other exploits spatial diversity among them effectively. This can largely solve the problems caused by shadowing, multipath fading and hidden node problem in spectrum sensing. Moreover it improves the probability of miss detection and the probability of false alarm. Cooperative spectrum sensing ([16], [24]) is called centralized, when a central unit gathers sensing data from the Cognitive Radios and identifies the spectrum usage. It is distributed if each local user uses the observations to make a local decision and sends this to the fusion center to make the final decision. Secondary users can either transmit a soft decision (summary statistic) or a hard decision [16]. Soft decisions provide better performance but at the cost of higher bandwidth consumption by the control channels between the Cognitive Radio and the fusion center. However as the number of cooperative users increases, hard decisions can perform as well [4].

An extensive survey of spectrum sensing methods is provided in [24]. One can use a fixed sample size (one shot) detectors or sequential detectors ([7], [10], [19], [24]). In fixed sample size detectors, the matched filter is optimal when there is complete knowledge of the primary signal. When the only a prior knowledge is about the noise power, then an energy detector is optimal in Neyman-Pearson criterion. However sequential detectors perform better. A recent survey is [12]. The sequential detectors can detect change or test a hypothesis. Sequential hypothesis testing finds out whether the primary is ON or OFF, while the sequential change point detection detects the point when the primary turns ON (or OFF). Sequential change detection is well studied (see [2], [10], [12], [14] and the references therein). However the optimal solution in the distributed setup is still not available. Sequential hypothesis testing ([5], [9], [19]) is useful when the status of the primary channel is known to change very slowly, e.g., detecting occupancy of a TV channel. Usage of idle TV bands by the Cognitive network is being targeted as the first application for cognitive radio. In this setup Walds' Sequential Probability Ratio Test (SPRT) [22] provides the optimal performance for a single node ([15], [23]). But the cooperative setup is not well explored.

We consider cooperative spectrum sensing using sequential hypothesis testing. SPRT is used at both the secondary nodes and the fusion center. This has been motivated by our previous algorithm DualCUSUM for change detection [7]. This algorithm is called DualSPRT and has been studied in ([8], [9] and [19]). As against [9] and [19], in [8] it has been analysed theoretically. Also in [8] this algorithm has been generalized to also include channel gain uncertainty. In [13] and a recent paper [6] (which we have noticed after our paper was accepted) asymptotically optimal decentralized sequential algorithms are obtained but do not consider fusion center noise. They also do not consider fading and receiver noise power uncertainties.

Although DualSPRT works well, it is not optimal. In this paper we improve over DualSPRT. Furthermore we introduce a new way of quantizing SPRT decisions of local nodes. We

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call this algorithm SPRT-CSPRT. We extend this algorithm to cover SNR uncertainties and fading channels. We also provide its theoretical analysis. We have seen via simulations that our algorithm works better than the algorithm in [13] and almost as well as the algorithm in [6] even when fusion center noise is not considered and the Multiple Access Channel (MAC) layer transmission delays are ignored in [6] and [13].

This paper is organised as follows. Section II describes the model. Section III starts with the DualSPRT algorithm. Then we provide SPRT-CSPRT and DualCSPRT algorithms developed in this paper. We compare their performance. Next we consider the receiver SNR uncertainty and slow fading channels. Section IV provides the theoretical analysis and compares to simulations. Section V concludes the paper.

II. Model

Consider a Cognitive Radio system with one primary transmitter and L secondary users. The L local nodes sense the channel to detect the spectral holes (i.e whether the primary is transmitting or not). The decisions made by the secondary users are transmitted to a fusion node via a MAC for it to make the final decision.

Let $X_{k,l}$ be the observation made at secondary user l at time k. We assume that $\{X_{k,l},\ k \geq 1\}$ are independent and identically distributed (i.i.d.) and that the observations are independent across Cognitive Radios. Using the detection algorithm based on $\{X_{n,l},\ n \leq k\}$ the secondary user l transmits $Y_{k,l}$ to the fusion node. We also assume that the secondary nodes are synchronised so that the fusion node receives $Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_k$, where $\{Z_k\}$ is i.i.d. zero mean Gaussian receiver noise with variance σ^2. The fusion center observes $\{Y_k\}$ and decides upon the hypothesis.

The observations $\{X_{k,l}\}$ depend on whether the primary is transmitting (Hypothesis H_1) or not (Hypothesis H_0):

$$X_{k,l} = \begin{cases} Z_{k,l}, & k = 1,2,\ldots,\ \textit{under } H_0, \\ h_l S_k + Z_{k,l}, & k = 1,2,\ldots,\ \textit{under } H_1, \end{cases} \tag{1}$$

where h_l is the channel gain of the l^{th} user, S_k is the primary signal and $Z_{k,l}$ is the noise at the l^{th} user at time k. We assume $\{Z_{k,l}, k \geq 1\}$ are i.i.d. . Let the fusion center makes a decision at time N. We assume that N is much less than the coherence time of the channel so that the slow fading assumption is valid. This means that h_l is random but remains constant during the spectrum sensing duration.

The general problem is to develop a distributed algorithm in the above setup which solves the problem:

$$\min E_{DD} \stackrel{\triangle}{=} E[N|H_i]\ , \tag{2}$$
$$\text{subject to } P_{FA} \leq \alpha$$

where H_i is the true hypothesis, $i = 0, 1$ and P_{FA} is the probability of false alarm, i.e., probability of making a wrong decision. We will separately consider $E[N|H_1]$ and $E[N|H_0]$. It is well known that for a single node case ($L = 1$) Wald's SPRT performs optimally in terms of reducing $E[N|H_1]$ and $E[N|H_0]$ for a given P_{FA}. If there is no communication cost or energy cost in transmitting data from the local nodes to the fusion node, then again we can realiably send data sensed by the local nodes to the fusion node and run SPRT at the fusion center. Otherwise, the optimal algorithm is not known. Motivated by the good performance of DualCUSUM in ([1], [7]) and the optimality of SPRT for a single node, we proposed DualSPRT in [8] and studied its performance. Now we modify DualSPRT to SPRT-CSPRT and DualCSPRT and we present the theoretical analysis.

III. Sequential Spectrum Sensing Algorithms

We first present DualSPRT which was introduced in our previous work [8].

A. DualSPRT Algorithm

1) Secondary node, l, runs SPRT algorithm,

$$W_{0,l} = 0$$
$$W_{k,l} = W_{k-1,l} + \log\left[f_{1,l}\left(X_{k,l}\right)/f_{0,l}\left(X_{k,l}\right)\right], k \geq 1 \tag{3}$$

where $f_{1,l}$ is the density of $X_{k,l}$ under H_1 and$f_{0,l}$ is the density of $X_{k,l}$ under H_0.

2) Secondary node l transmits a constant b_1 at time k if $W_{k,l} \geq \gamma_1$ or transmits b_0 when $W_{k,l} \leq \gamma_0$, i.e.,

$$Y_{k,l} = b_1 1_{\{W_{k,l} \geq \gamma_1\}} + b_0 1_{\{W_{k,l} \leq \gamma_0\}}$$

where $\gamma_0 < 0 < \gamma_1$ and 1_A denotes the indicator function of set A. Parameters $b_1, b_0, \gamma_1, \gamma_0$ are chosen appropriately.

3) Physical layer fusion is used at the Fusion Centre, i.e., $Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_k$, where Z_k is the i.i.d. noise at the fusion node.

4) Finally, Fusion center runs SPRT:

$$F_k = F_{k-1} + \log\left[g_1\left(Y_k\right)/g_0\left(Y_k\right)\right], \quad F_0 = 0, \tag{4}$$

where g_0 is the density of $Z_k + \mu_0$ and g_1 is the density of $Z_k + \mu_1$, μ_0 and μ_1 being design parameters.

5) The fusion center decides about the hypothesis at time N where

$$N = \inf\{k : F_k \geq \beta_1 \ \textit{or}\ F_k \leq \beta_0\}$$

and $\beta_0 < 0 < \beta_1$. The decision at time N is H_1 if $F_N \geq \beta_1$; otherwise H_0.

B. SPRT-CSPRT Algorithm

In DualSPRT given above, observations to the fusion center are not always identically distributed. Till the first transmission from secondary nodes, these observations are i.i.d. $\sim \mathcal{N}(0, \sigma^2)$ where $\mathcal{N}(a, b)$ is the Gaussian pdf with mean a and variance b. But after the transmission from the first local node and till the transmission from the second node, they are i.i.d. Gaussian with another mean and same variance σ^2. Thus the observations at the fusion center are no longer i.i.d. . Since the optimality of SPRT is known for i.i.d. observations ([23], [15]), DualSPRT is not optimal.

The following heuristic arguments provide the motivation of the proposed modifications to DualSPRT. A sample path of the fusion center SPRT under the hypothesis H_1 is given in Figure 1. If the SPRT sum defined in (4) goes below zero it delays in crossing the positive threshold β_1. Hence if we keep SPRT sum at zero whenever it goes below zero, it reduces E_{DD}. This happens in CUSUM ([14], [15]). Similarly one can use a CUSUM type algorithm under H_0. Thus we obtain the following algorithm,

Steps (1)-(3) are same as in DualSPRT. The steps (4) and (5) are replaced by

4) Fusion center runs two algorithms:

$$F_k^1 = (F_{k-1}^1 + \log\left[g_1\left(Y_k\right)/g_0\left(Y_k\right)\right] + D_1)^+ \quad (5)$$

$$F_k^0 = (F_{k-1}^0 + \log\left[g_1\left(Y_k\right)/g_0\left(Y_k\right)\right] + D_0)^-, \quad (6)$$

$F_0^1 = 0, F_0^0 = 0$, where $(x)^+ = \max(0, x)$ and $(x)^- = \min(0, x)$. D_1 and D_0 are appropriately chosen constants to introduce bias to the drift.

5) The fusion center decides about the hypothesis at time N where

$$N = \inf\{k : F_k^1 \geq \beta_1 \textit{ or } F_k^0 \leq \beta_0\}$$

and $\beta_0 < 0 < \beta_1$. The decision at time N is H_1 if $F_N^1 \geq \beta_1$, otherwise H_0.

Under H_1, (5) has a positive drift and hence it approaches the threshold β_1 quickly, but under H_0, (5) will most probably be hovering around zero. Similarly under H_0, (6) moves towards β_0, but under H_1 will be mostly around zero. This means that P_{FA} for this algorithm is expected to be less compared to DualSPRT.

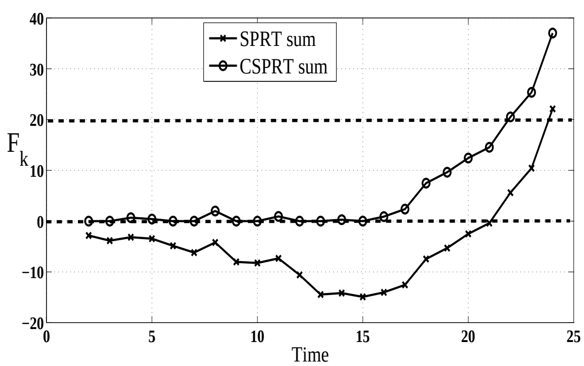

Fig. 1. Sample Path of F_k under SPRT Sum and CSPRT Sum for $\gamma_1 = 8$, $\beta_1 = 20$, $\mu_1 = 1$ and $\mu_0 = -1$

We consider one more improvement. When a local Cognitive Radio SPRT sum crosses its threshold, it transmits b_1/b_0. This node transmits till the fusion center SPRT crosses the threshold. If it is not a false alarm, then its SPRT sum keeps on increasing (decreasing). But if it is a false alarm, then the sum will eventually move towards the other threshold. Hence instead of transmitting b_1/ b_0 the Cognitive Radio can transmit a higher / lower value in an intelligent fashion. This should improve the performance. Thus we modify the step (3) in DualSPRT as follows. Secondary node l transmits a constant from $\{b_1^1, b_2^1, b_3^1, b_4^1\}$ at time k if $W_{k,l} \geq \gamma_1$ or transmits from $\{b_1^0, b_2^0, b_3^0, b_4^0\}$ when $W_{k,l} \leq \gamma_0$, as follows :

$$Y_{k,l} = \begin{cases} b_1^1 \text{ if } W_{k,l} \in \ [\gamma_1, \gamma_1 + \Delta_1), \\ b_2^1 \text{ if } W_{k,l} \in \ [\gamma_1 + \Delta_1, \gamma_1 + 2\Delta_1), \\ b_3^1 \text{ if } W_{k,l} \in \ [\gamma_1 + 2\Delta_1, \gamma_1 + 3\Delta_1), \\ b_4^1 \text{ if } W_{k,l} \in \ [\gamma_1 + 3\Delta_1, \infty), \\ b_1^0 \text{ if } W_{k,l} \in \ [\gamma_0, \gamma_0 - \Delta_0), \\ b_2^0 \text{ if } W_{k,l} \in \ [\gamma_0 - \Delta_0, \gamma_0 - 2\Delta_0), \\ b_3^0 \text{ if } W_{k,l} \in \ [\gamma_0 - 2\Delta_0, \gamma_0 - 3\Delta_0), \\ b_4^0 \text{ if } W_{k,l} \in \ [\gamma_0 - 3\Delta_0, -\infty). \end{cases} \quad (7)$$

where Δ_1 and Δ_0 are the parameters to be tuned at the Cognitive Radio. The expected drift under H_1 (H_0) is a good choice for Δ_1 (Δ_0).

We call the algorithm with the above two modifications as SPRT-CSPRT (with 'C' as an indication about the motivation from CUSUM).

If we use CSPRT at both the secondary nodes and the fusion center with the proposed quantisation methodology (we call it DualCSPRT) it works better as we will show via simulations in Section III C. In the Section IV we will theoretically analyse SPRT-CSPRT. As the performance of DualCSPRT (Figure 2) is nearer to that of SPRT-CSPRT, we analyse only SPRT-CSPRT.

C. Performance Comparison

Throughout the paper we use $\gamma_1 = -\gamma_0 = \gamma$, $\beta_1 = -\beta_0 = \beta$ and $\mu_1 = -\mu_0 = \mu$ for the simplicity of the simulation and analysis.

We apply DualSPRT, SPRT-CSPRT and DualCSPRT on the following example and compare their E_{DD} for various values of P_{FA}. We assume that the pre-change distribution f_0 and the post change distribution f_1 are Gaussian with different means. This type of modelling is relevant when noise and interference are log-normally distributed [21]. This is also a useful model when $X_{k,l}$ is the sum of energy of a large number of observations at the secondary node at low SNR.

Parameters used for simulation are as follows: There are 5 nodes ($L = 5$), $f_{0,l} \sim \mathcal{N}(0, 1)$, for $1 \leq l \leq L$. Primary to secondary channel gains are 0, -1.5, -2.5, -4 and -6 dB respectively (the corresponding post change means of Gaussian distribution with variance 1 are 1, 0.84, 0.75, 0.63 and 0.5). We assume $Z_k \sim \mathcal{N}(0, 5)$ and drift of DualSPRT and SPRT-CSPRT at the fusion center is taken as $2\mu Y_k$, with μ being 1. We also take $D_0 = D_1 = 0$, $\{b_1^1, b_2^1, b_3^1, b_4^1\} = \{1, 2, 3, 4\}$, $\{b_1^0, b_2^0, b_3^0, b_4^0\} = \{-1, -2, -3, -4\}$ and b_1=$-b_0$=1(for DualSPRT). Parameters γ and β are chosen from a range of values to achieve a particular P_{FA}. Figure 2 provides the E_{DD} and P_{FA} via simulations. We see a significant improvement in E_{DD} compared to DualSPRT. The difference increases as P_{FA} decreases. The performance under H_0 is similar. Performance comparisons with the asymptotically optimal decentralized sequential algorithms which do not consider fusion center noise (DSPRT [6], Mei's SPRT [13]) are given in Figure 3. Note that DualSPRT and SPRT-CSPRT include fusion center noise. Here we take $f_{0,l} \sim \mathcal{N}(0, 1)$, $f_{1,l} \sim \mathcal{N}(1, 1)$ for $1 \leq l \leq L$ and $Z_k \sim \mathcal{N}(0, 1)$. We find that the SPRT-CSPRT's performance close to that of DSPRT and better than

Mei's SPRT. Similar comparisons were obtained with other data sets.

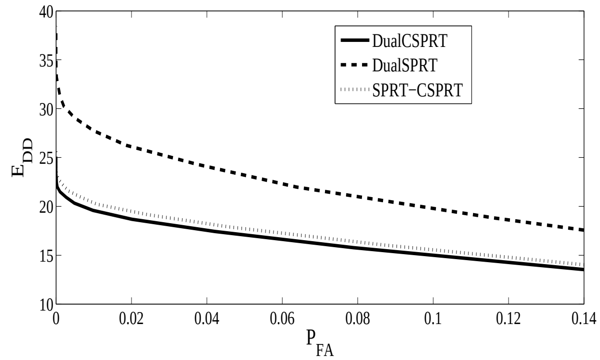

Fig. 2. Comparison among DualSPRT, SPRT-CSPRT and DualCSPRT for different SNR's between the primary and the secondary users, under H_1

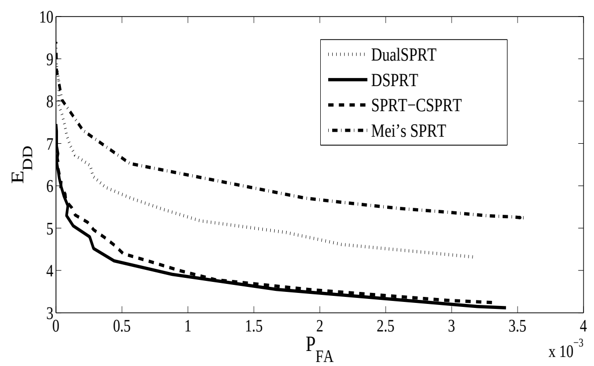

Fig. 3. Comparison among DualSPRT, SPRT-CSPRT, Mei's SPRT and DSPRT under H_1

D. Unknown Received SNR and Fading

In this section, now we consider the following setup. We use energy detector at the Cognitive Radios, i.e, the observations $X_{k,l}$ are a summation of energy of past N_1 observations received by the l^{th} Cognitive Radio node. Then if N_1 is reasonably large, $X_{k,l}$ are approximately Gaussian. If the received SNR at the Cognitive Radio is not known then the hypothesis testing problem can be approximated as a change in mean of Gaussian distributions problem, where the mean θ_1 under H_1 is not known. For this case in [8] we used composite sequential hypothesis testing proposed in [11] at the secondary nodes and used SPRT at the fusion node. This was called GLR-SPRT [8]. Here, to take the advantage of CSPRT at the fusion node and the new quantisation technique we modify GLR-SPRT [8] to GLR-CSPRT with appropriate local quantisation. Thus the secondary node's hypothesis testing problem, SPRT, stopping criteria and decision are modified as follows,

$$H_0 : \theta = \theta_0 \ ; \ H_1 : \theta \geq \theta_1 \ . \quad (8)$$

where $\theta_0 = 0$ and θ_1 is appropriately chosen,

$$W_{n,l} = max\left[\sum_{k=1}^{n} \log \frac{f_{\hat{\theta_n}}(X_k)}{f_{\theta_0}(X_k)}, \sum_{k=1}^{n} \log \frac{f_{\hat{\theta_n}}(X_k)}{f_{\theta_1}(X_k)}\right] \ , \quad (9)$$

$$N = \inf\{n : W_{n,l} \geq g(cn)\} \ , \quad (10)$$

where $g()$ is a time varying threshold and c is the cost assigned for each observation. Its approximate expression is given in [11]. Also for Gaussian f_0 and f_1, $\theta \in [a_1, a_2]$ and S_n as the summation of observations $X_{k,l}$ upto time n, $\hat{\theta}_n = max\{a_1, min[S_n/n, a_2]\}$. At time N decide upon H_0 or H_1 according as $\hat{\theta}_N \leq \theta^* \ or \ \hat{\theta}_N \geq \theta^*$, where θ^* is obtained by solving $I(\theta^*, \theta_0) = I(\theta^*, \theta_1)$, and $I(\theta, \lambda)$ is the Kullback-Leibler information number. Here, as the threshold is a time varying and decreasing function, the quantisation (7) is changed in the following way: if $\hat{\theta}_N \geq \theta^*$

$$Y_{k,l} = \begin{cases} b_1^1 \text{ if } W_{k,l} \in \ [g(kc), g(kc3\Delta)), \\ b_2^1 \text{ if } W_{k,l} \in \ [g(kc3\Delta), g(kc2\Delta)), \\ b_3^1 \text{ if } W_{k,l} \in \ [g(kc2\Delta), g(kc\Delta)), \\ b_4^1 \text{ if } W_{k,l} \in \ [g(kc\Delta), \infty). \end{cases} \quad (11)$$

If $\hat{\theta}_N \leq \theta^*$ we will transmit from $\{b_1^0, b_2^0, b_3^0, b_4^0\}$ under the same conditions. Here Δ is a tuning parameter and $0 \leq 3\Delta \leq 1$. The choice of θ_1 in (8) affects the performance of $E[N|H_0]$ and $E[N|H_1]$ for the algorithm (9)-(10). As θ_1 increases, $E[N|H_0]$ decreases and $E[N|H_1]$ increases.

The performance comparison of GLR-SPRT and GLR-CSPRT for the example in Section III C (with $Z_k \sim \mathcal{N}(0, 1)$) is given in Figure 4 and Figure 5. Here $\Delta = 0.25$. As the performance under H_1 and H_0 are different, we give the values under both. We can see that GLR-SPRT is always inferior to GLR-CSPRT. For E_{DD} under H_1, interestingly GLR-CSPRT have lesser values than that of SPRT-CSPRT for $P_{FA} > 0.02$ (note that SPRT-CSPRT has complete knowledge of the SNRs), while under H_0 it has higher values than SPRT-CSPRT.

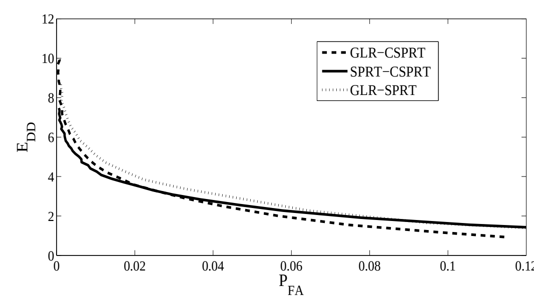

Fig. 4. Comparison among SPRT-CSPRT, GLR-SPRT and GLR-CSPRT for different SNR's between the primary and the secondary users under H_1

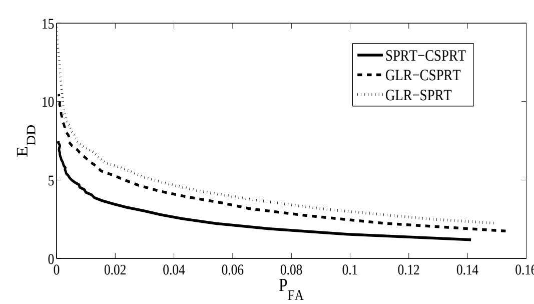

Fig. 5. Comparison among SPRT-CSPRT, GLR-SPRT and GLR-CSPRT for different SNR's between the primary and the secondary users under H_0

The above scenario can also occur if the fading channel gain h_l is not known to the Cognitive node l. Then under slow fading with Rayleigh distribution and using energy detector at the Cognitive Radios, $f_{0,l} \sim \mathcal{N}(0, \sigma^2)$ and $f_{1,l} \sim \mathcal{N}(\theta, \sigma^2)$ where θ is random with exponential distribution and σ^2 is the variance of noise. Here we use the GLR-CSPRT with the composite sequential hypothesis given in (8). The parameter θ_1 is chosen as the median of the distribution of θ, such that $P(\theta \geq \theta_1) = 1/2$. This seems a good choice for θ_1 to compromise between $E[N|H_0]$ and $E[N|H_1]$. We use the example given in Section III C with $Z_k \sim \mathcal{N}(0, 1)$ and $\theta \sim exp(1)$. Table I provides comparison of DualSPRT, GLR-SPRT and GLR-CSPRT. Notice that the comment given for E_{DD} for Figure 4 is also valid here.

Hyp	E_{DD}	$P_{FA} = 0.1$	$P_{FA} = 0.07$	$P_{FA} = 0.04$
H_1	DualSPRT	1.74	1.948	2.728
H_1	GLR-SPRT	1.62	3.533	9.624
H_1	GLR-CSPRT	0.94	1.004	4.225
H_0	DualSPRT	1.669	1.891	2.673
H_0	GLR-SPRT	3.191	3.849	4.823
H_0	GLR-CSPRT	2.615	3.192	4.237

TABLE I
COMPARISON AMONG DUALSPRT, GLR-SPRT AND GLR-CSPRT WITH SLOW FADING BETWEEN THE PRIMARY AND THE SECONDARY USERS

IV. PERFORMANCE ANALYSIS OF SPRT-CSPRT

E_{DD} and P_{FA} analysis is same under H_1 and H_0. Hence we provide analysis under H_1 only.

A. P_{FA} Analysis

Let P_0 and P_1 denote the probability measure under H_0 and H_1 respectively. Between each change of drift (which occurs due to the change in number of Cognitive Radios transmitting to the fusion node and due to the change in the value transmitted according to the quantisation rule (7)) at the fusion center, under H_1, (5) has a positive drift and behaves approximately like a normal random walk. (6) also has a positive drift, but due to the $\min$ in its expression it will stay around zero and as the event of crossing negative threshold is rare (6) becomes a reflected random walk between each drift change. Similarly under H_0, (5) and (6) become reflected random walk and normal random walk respectively. The false alarm occurs when the reflected random walk crosses its threshold. Under H_1, let

$$\tau_\beta \stackrel{\triangle}{=} \inf\{k \geq 1 : F_k^0 \leq -\beta\}. \quad (12)$$

We call τ_β the first passage time at the fusion center. Let $\tau_{\gamma,l}$ be the first passage time to threshold γ by the l^{th} node. Let t_k be the k^{th} order statistics of L i.i.d. random variables. Then P_{FA} at the fusion node, when H_1 is the true hypothesis is given by,

$$P_{H_1}(False\ alarm) = P_{H_1}(False\ alarm\ before\ t_1) \quad (13)$$
$$+P_{H_1}(False\ alarm\ between\ t_1 and\ t_2)$$
$$+P_{H_1}(False\ alarm\ between\ t_2\ and\ t_3) + \ldots.$$

The main contribution to P_{FA} comes from the first term as γ increases.

$$P_{H_1}(FA\ before\ t_1) = \sum_{k=1}^{\infty} P(\tau_\beta \leq k, k < t_1) = \sum_{k=1}^{\infty} P(\tau_\beta \leq k | k < t_1) P(t_1 > k) \quad (14)$$

In the following we compute $P_0\{\tau_\beta > x | \tau_\beta < t_1\}$ and $P[t_1 > k]$. It is shown in [17] that,

$$\lim_{\beta \to \infty} P_0\{\tau_\beta > x | \tau_\beta < t_1\} = \exp(-\lambda_\beta x), x > 0. \quad (15)$$

By finding solution to the integral equation obtained via renewal arguments [18], we can obtain the mean $1/\lambda_\beta$ of first passage time, τ_β (as done in [1], [2]). Let $L(s)$ be the mean of τ_β with $F_0^0 = s$ and $S_k = \log\left[g_1(Y_k)/g_0(Y_k)\right] + D_0$. From the renewal arguments, by conditioning on $S_0 = z$:

$$L(s) = F_S(-s)(L(0)+1) + \int_{-s}^{\beta - s} (L(s+z)+1) dF_S(z) dz + P[S > \beta - s]. \quad (16)$$

where F_S is the distribution of S_k before the first transmission from the local nodes. By solving these equations numerically, we get $\lambda_\beta = 1/L(0)$.

Next we consider the distribution of t_1. SPRT $\{W_{k,l}, k \geq 0\}$ is a random walk at each secondary node l. We assume $f_{0,l} \sim \mathcal{N}(0, \sigma_l^2)$ and $f_{1,l} \sim \mathcal{N}(\theta_l, \sigma_l^2)$, where θ_l is the post change mean and σ_l^2 is the variance for l^{th} Cognitive Radio. Let mean and variance of the drift of l^{th} Cognitive Radio be $\delta_l = E_{H_1}[\log\left(f_1(X_{k,l})/f_0(X_{k,l})\right)]$, $\Sigma_l^2 = Var_{H_1}[\log\left[f_1(X_{k,l})/f_0(X_{k,l})\right]]$ respectively. We know $\delta_l > 0$. The time $\tau_{\gamma,l}$ for $W_{k,l}$ at each local node l to cross the threshold γ satisfies $E[\tau_{\gamma,l}] \sim \gamma/\delta_l$ for large values of γ (needed for small P_{FA}). Then by central limit theorem we can show that at each node l

$$\tau_{\gamma,l} \sim \mathcal{N}\left(\frac{2\sigma_l^2\gamma}{\theta_l^2}, \frac{8\sigma_l^4\gamma}{\theta_l^4}\right). \quad (17)$$

Thus now (14) equals

$$\approx \sum_{k=1}^{\infty} (1 - e^{-\lambda_\beta k}) \prod_{l=1}^{L} (1 - \Phi_{\tau_{\gamma,l}}(k))$$

where $\Phi_{\tau_{\gamma,l}}$ is the Cumulative Distribution Function of $\tau_{\gamma,l}$, obtained from the Gaussian approximation (17).

Table II provides comparison of P_{FA} via simulation and analysis.

$P_{FA} Sim.$	$P_{FA} Anal.$	$E_{DD} Sim.$	$E_{DD} Anal.$
0.0072	0.0065	33.1585	31.7624
0.00675	0.00613	26.8036	24.9853
0.01675	0.01624	30.0817	29.1322

TABLE II
COMPARISON OF E_{DD} AND P_{FA} OBTAINED VIA ANALYSIS AND SIMULATION UNDER H_1

B. E_{DD} Analysis

In this section we compute E_{DD} theoretically. t_i, i^{th} order statistics of L random variables $\tau_{\gamma,l}, 1 \leq l \leq L$, is the first time at which i local nodes are transmitting. Mean of t_i can be find out from the method explained in [3], for finding k^{th} central moment of non i.i.d. L^{th} order statistics.

Between t_i and t_{i+1} the drift at the fusion center is not necessarily constant because there are four thresholds (each corresponds to different quantizations) at the secondary node. The transmitted value changes after crossing each threshold, $b_1 \to b_2 \ldots \to b_4$. Let $t_i^j, 1 \leq j \leq 3$ be the time points at which a node changes the transmitting values from b_j to b_{j+1} between t_i and t_{i+1}. We assume that the probability of false alarm at the local nodes, P_{fa} is very small. Also with a high probability the secondary node with the lowest mean in (17) will transmit first, the node with the second lowest mean will transmit second and so on. In the following we will make computations under this approximations. The time difference between t_i^{jth} and t_{i+1}^{jth} transmission can be calculated if we take the second assumption (=Δ_1/δ_l). We know $E[t_i]$ for every i from an argument given earlier. Suppose l^{th} node transmits at t_i^{th} instant and if $E[t_i] + \Delta_1/\delta_l < E[t_{i+1}]$ then $E[t_i^1] = E[t_i] + \Delta_1/\delta_l$. Similarly if $E[t_i^1] + \Delta_1/\delta_l < E[t_{i+1}]$ then $E[t_i^2] = E[t_i^1]+\Delta_1/\delta_l$ and so on. Let us represent the sequence $t = \{t_1, t_1^1, t_1^2, t_1^3, t_2, ..., t_5^5\}$ (entry only for existing ones by the above criteria) by $T = \{T_1, T_2, T_3, ...\}$.

Let μ_k be the mean drift at the fusion center between T_k and T_{k+1}. Thus T_k's are the transition epochs at which the fusion center drift changes from μ_{k-1} to μ_k. Also let $\bar{F}_k = E[F_{T_k - 1}]$ be the mean value of F_k just before the transition epoch T_k. With the assumption of the very low P_{fa} at the local nodes and from the knowledge of the sequence t we can easily calculate μ_k for each T_k. Similarly $\bar{F}_{k+1} = \bar{F}_k + \mu_k(E[T_{k+1}] - E[T_k])$. Then,

$$E_{DD} \approx E[T_j] + \frac{\beta - \bar{F}_j}{\mu_j} \tag{18}$$

where

$$j = min\{i : \mu_i > 0 \ and \ \frac{\beta - \bar{F}_i}{\mu_i} < E[T_{i+1}] - E[T_i]\}.$$

Table II provides the simulation and corresponding analysis values. We used the same set-up as in Section III C (with $Z_k \sim \mathcal{N}(0, 1)$)

V. Conclusion

We consider the problem of cooperative spectrum sensing in this paper. We provide improved algorithms SPRT-CSPRT and DualCSPRT over a recent algorithm DualSPRT. We show that these algorithms can provide significant improvements (simulation studies show that the improvement is over 25 %). Surprisingly, to develop the improved algorithm we use CUSUM algorithm used for detection of change rather than SPRT which is optimal for hypothesis testing for a single node. This happens because SPRT lets the likelihood ratio stray into wrong direction while CUSUM clips it at zero. We provide theoretical analysis of SPRT-CSPRT and compare to simulations. Interestingly we found that the performance of the proposed algorithm is better than one and close to another asymptotically optimal algorithms for decentralized sequential detection without fusion center noise. We further extend our algorithms to cover the case of unknown SNR and channel fading and obtain satisfactory performance compared to perfect channel state information case.

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